Mutual Information Based Registration of SAR Images

Hua Xie, Leland E. Pierce, Fawwaz T. Ulaby

Radiation Laboratory, Electrical Engineering and Computer Science Dept.
The University of Michigan, Ann Arbor, MI 48109-2122, USA
Tel: (734) 763-3157 FAX: (734) 647-2106 E-mail: ulaby@eecs.umich.edu

ABSTRACT

Interpolation artifacts caused by the discrete nature of digital images and interpolation kernels have been previously reported in the literature for mutual information based image registration. Unfortunately they become pronounced in the case of SAR (Synthetic Aperture Radar) image registration, exacerbated by speckle noise. We analyzed the two widely used interpolation algorithms in image registration, namely bilinear interpolation and partial volume interpolation. According to simulation results, we found that the former algorithm produces an extremely nonsmooth metric function, whereas the latter method introduces spurious global optimum at some position of misalignment, instead of the position of perfect registration, under certain circumstances. To make the mutual information criterion useful for the application of SAR image registration, we proposed two pre-processing steps including speckle reduction and an appropriate selection of the number of bins for histogram construction. Simulation results indicate that, by this effort, interpolation artifacts in the mutual information function can be significantly suppressed, and as a consequence, accurate registration is allowed. In conjunction with the two proposed pre-processing steps, we implemented a multiscale elastic registration algorithm. An example of registering a pair of JERS-1 (L-band, HH-polarization) and RADARSAT (C-band, HH-polarization) images verified the potential of this algorithm.

1. INTRODUCTION

Mutual information provides a means to measure statistical dependence between two random variables or the amount of information that one variable contains about the other. Mutual information between two random variables $X$ and $Y$, denoted by $MI(X,Y)$, is related to entropy by

$$MI(X,Y) = H(X) + H(Y) - H(X,Y)$$  \hspace{1cm} (1)

Image registration refers to the process of spatially aligning a pair of images through a geometrical transformation performed on one of the images. Given two images, one is called the reference image $I_R$ and the other is called the floating image $I_F$, along with a similarity function which quantifies the quality of alignment between the two images, the objective of image registration is to find the transformation that maximizes the similarity function between $I_R$ and the transformed floating image. In general, a specific image registration method is fully characterized by four main components, namely feature space, search space, similarity metric and search strategy. The basic concept behind the use of mutual information for registering images $I_R$ and $I_F$ is that mutual information (similarity metric) calculated from intensity values (feature space) in the two images will reach its maximum when the images are geometrically perfectly aligned.

Since Collignon et al. [1] and Viola et al. [2] independently proposed mutual information as a quality measure to register multi-modal medical images in 1995, mutual information based image registration has generated tremendous interest because of its advantages of being fully automatic, robust and efficient. At present, to the best of our knowledge, there is no single article comprehensively devoted to SAR image registration where mutual information is involved as a similarity function. This motivates us to investigate the potential of mutual information for SAR image registration, especially the robustness and reliability of this metric under the adversity of speckle noise present in SAR imagery. In Section 2 we consider implementation issues of mutual information based SAR image registration and then we devise an efficient multisresolution registration procedure using mutual information. We present some experimental results in Section 3 and offer our conclusions in Section 4.

2. IMPLEMENTATION FOR MUTUAL INFORMATION BASED SAR IMAGE REGISTRATION

2.1. Transformation

In general, rigid and affine transformations are by no means sufficient for most remote sensing applications as they cannot capture different local deformation caused by the variations in sensor platform (altitude, attitude, and velocity), earth curvature and earth rotation. We consider thin plate spline (TPS) elastic warping [3] for images in remote sensing applications. In 2-D image registration, given two sets of ground control points (GCPs), $p_i = (x_i, y_i)$ and $q_i = (x'_i, y'_i), i = 1, 2, ..., n$, the thin plate spline transformation $f = (f^x, f^y) : \mathbb{R}^2 \to \mathbb{R}^2$ mapping function are formulated as

$$f^x(x,y) = a_0^x + a_1^x x + a_2^x y + \sum_{i=1}^{n} w_i f_i U(|p_i - (x,y)|)$$, \hspace{1cm} (2)

$$f^y(x,y) = a_0^y + a_1^y x + a_2^y y + \sum_{i=1}^{n} w_i f_i U(|p_i - (x,y)|)$$, \hspace{1cm} (3)
where $U(r) = r^2 \log(r^2)$ is the biharmonic equation, in which $r$ is the distance in Cartesian coordinate $r = \sqrt{x^2 + y^2}$.

2.2. Probability Density Function Estimation

The underlying probability density functions (pdf) of the reference and floating images are estimated by using histograms in this study. To estimate the pdf with high fidelity, the number of intensity bins is very important. In [4], the optimal histogram bin width $W$ was derived based on minimizing the integrated mean squared error between the true density $f$ and the histogram estimate. It can be calculated as

$$W = \left\{ 6\int_{-\infty}^{\infty} f'(x)^2 \, dx \right\}^{1/3} N^{-1/3}$$

(4)

where $f'(x)$ is the derivative of the true density $f$, and $N$ is the sample size. As indicated in (4), the optimal choice for the bin width requires knowledge of the true underlying density $f$, which would not be the case in most real applications. Considering the bin number in the context of image registration, there is a trade-off between registration accuracy and the sensitivity of the mutual information to the change of transformation parameters. Our simulation results show that it is not critically necessary to determine the optimal number of bins for SAR image registration. In practice, we find that the bin number of the joint histogram, that on average maintains at least one entry per bin in the joint histogram, works very well as long as used in conjunction with speckle reduction. For images with discrete intensities, it is also advised not to have the number of bins exceed the intensity levels.

2.3. Interpolation Artifacts and Speckle Reduction

Because of the discrete nature of digital images, the transformed position of the original floating image will not fall on the grid which the reference image sits on for each transformation, except for shifts over integer grid points or rotations about multiples of 90°; hence, interpolation is necessary in order to obtain the resampling at those non-grid positions. Bilinear interpolation (trilinear interpolation in the 3-D case) and partial volume (PV) interpolation [1] are two popularly used interpolation processes. Interpolation artifacts of local optima in the mutual information function have been studied by Pluim et al. [5]. It is expected that artifacts become pronounced due to the presence of speckle noise in SAR image registration. To investigate the degree of deterioration of interpolation artifacts caused by speckle noise, we simulate a reference SAR image and a floating SAR image using the original Lena image (256 × 256, 8 bits), the former of which is generated by multiplying speckle noise with the original Lena image, whereas the latter is a transformed Lena image corrupted by speckle noise. The simulation is limited to a rigid transformation for simplicity. The speckle noise in intensity format is simulated independently and then multiplied with the original and the transformed images. Examples of the reference and floating images are displayed in Fig. 1 for the case of single-look speckle noise.

Figure 1: Examples of the reference and floating Lena images

Figs. 2(a) and 2(b) depict the 2-D mutual information as a function of rotation and translation along the x-axis for the Lena images corrupted by single-look speckle noise as shown in Fig. 1, for bilinear interpolation and PV interpolation respectively. The desired optimal should occur at $\theta = 5^\circ$ and $t_x = 0$ for both Figs. 2(a) and 2(b). Weighted averaging, inherent to bilinear interpolation, functions like a low-pass filter at non grid-aligning transformations. When the images to be registered are noisy, noise reduction may reduce the dispersion of the joint histogram and result in an increased value of mutual information, even at a misalignment transformation. This explains why there is a local minimum at rotation angle $\theta = 0^\circ$ in Fig. 2(a) where bilinear interpolation is used. As far as PV interpolation is concerned, our simulation results show that it may introduce an undesirable global maximum into the mutual information function if the number of bins used for the histogram is large. Artifacts in this method stem from PV interpolation's multiple-entry histogram updating scheme. When two images are aligned on grids, only one histogram entry will be updated with an increment of one, as the rest of the PV weights are all zero. It follows that the joint histogram seems less dispersed and mutual information will be consequently higher. In the case of translation, interpolation artifacts become increasingly obvious, as illustrated in Figs. 2(a) and 2(b), with the periodic pattern of local optima at $\theta = 0^\circ$. As opposed to the local minima induced by bilinear interpolation, PV interpolation presents local maxima at translation of grid points. When the pixel sizes of the two images are not related to each other by a simple integer scale, a scale change would make interpolation artifacts less severe.

Decreasing the number of bins is equivalent to performing low-pass filtering on noisy images. As the number of joint histogram bins decreases, the mutual information function based on bilinear interpolation becomes smoother, whereas the false global maximum in the mutual information function based on partial volume interpolation turns into a local maximum. All these changes are beneficial to accurate image registration, which leads to the conclusion that an appropriate choice of the number of bins is very important. It is noticed that the
noise level also has an impact on the seriousness of interpolation artifacts. Simulations demonstrate that as the number of looks increases, interpolation artifacts are reduced accordingly. We apply the wavelet despeckling algorithm developed in [6] to suppress noise in the reference and floating images before computing mutual information. Shown in Figs. 2(c) and 2(d) are drastically improved surface functions as compared to their respective counterparts in Figs. 2(a) and 2(b). These examples suggest that a significant reduction of interpolation artifacts can be achieved through speckle reduction and a proper choice of the number of bins. It follows that these two steps are crucial to the overall registration accuracy, especially for SAR images with high-level of noise.

![Figure 2: 2-D mutual information function around the registered position for despeckled single-look Lena Images](image)

**2.4. Optimization**

Even though despeckling and proper binning significantly reduce the interpolation artifacts, they are incapable of completely removing local optima in the mutual information function for SAR images with high speckle noise levels. This statement is supported by Figs. 2(c) and 2(d), neither of which is totally free of interpolation artifacts. Care must be used when applying local optimization algorithms. In this study, we apply the multi-dimensional Nelder-Mead simplex method. To accelerate the process as well as to avoid having the algorithm getting trapped at local optima, we can provide a reasonable estimate of the transformation as a starting point to the optimization algorithm. Basically, human operators initiate the registration process by roughly selecting a few GCPs in the image pair, and let these GCPs drive the optimization process.

The number of GCPs can be as low as required in order to determine the transformation type of interest.

**2.5. Multiresolution Elastic Image Registration**

We adopt the multiscale elastic registration algorithm developed by Periaswamy et al. in [7] where transformation parameters are tuned in a coarse-to-fine fashion. Instead of using the correlation coefficient between the Fourier transforms of the two images as proposed in [7], we apply mutual information as the similarity measure. At level one, a rigid transformation is assumed between the two images being registered. A human operator collects GCPs to determine the transformation parameters which are then used as the initial guess to feed the multi-dimensional Nelder-Mead simplex method. Based on the optimization results, the original floating image is rigidly transformed to a new floating image, with the latter the process progresses to the next level. At level two, the two images are segmented into quadruple subimages and rigid transformation is continuously assumed between each pair of corresponding subimages. Then, with mutual information as a quality measure, the optimal rigid transformation is sought for each pair of subimages. For each block of the floating image, five GCPs are collected along with their corresponding points in the block of the reference image. The five GCPs in each block of the floating image consist of one center point and four points that are half-block length away from the center on its left, right, above and below. Once four sets of GCPs in all blocks are collected, a TPS model is derived to refine the global transformation. Next, the resulting TPS model is used to warp the current floating image and yield a new one. In principle, at each level, three tasks are implemented: quadruple segmentation, local rigid alignment and global elastic transformation. The entire procedure can stop when the termination criteria are met.

As the process of segmenting images into quadruple blocks continues, the dimension of each block is progressively decreased by a factor of 2, along both the x and y axes. According to (4), when the image size is decreased by a factor of four, the number of bins should be decreased by a factor of $(4)^{1/3} = 1.6$ provided the underlying statistics have not changed significantly. We choose a factor of 2 for simplicity. Experimental results indicate that this criterion is good in practice.

**3. EXPERIMENTAL RESULTS AND DISCUSSION**

The overall performance of the hierarchical elastic registration algorithm is tested on a pair of JERS-1 and RADARSAT images which cover a portion of Manaus in the Amazon basin. Both images are 3-looks. The pixel spacing of the JERS-1 data is 12.5m×12.5m, whereas the pixel spacing of the RADARSAT data is 50m×50m. Since this is only a feasibility study on SAR images, we first subsample the JERS-1 image to the same pixel spacing of the RADARSAT. By decreasing one degree of freedom in the transformation parameters, the registration problem can be simplified. After subsampling, the two
images are both of dimensions 600×600.

Prior to registration, both images are filtered by the wavelet despeckling algorithm developed in [6]. In this experiment, the multi-resolution process stops at level three. The histogram bin number used at level one, two and three is 128×128, 64×64 and 32×32 respectively. Figs. 3(a) and 3(b) present the original reference and floating images respectively. Fig. 3(c) is the pseudo-color composite of the reference image and the automatically registered floating image. GCP-based manual registration is also performed for the purpose of comparison. Nine pairs of GCPs are collected across the two original images, and the second order polynomial model is used to determine the transformation. The final result of manual registration is presented in Fig. 3(d). Visual inspection indicates that the accuracy of the automatic registration is very close to that of the manual registration. Quantitatively, the mutual information between the reference image and the registered floating image is 0.304 for the manual registration, and 0.249, 0.310 and 0.305 at the three levels of the automatic registration, respectively. Compared with level one in the case of automatic registration, the mutual information at level two increased by about 25%, indicating a high degree of improvement in registration accuracy. However, the mutual information at level three was not able to increase progressively. There are mainly two reasons to explain this fact. The first is related to interpolation artifacts. The level-two registration is already very close to the ground truth. Ideally, at level-three, only small perturbation around the position of $\theta = 0^\circ$, $t_x = 0$, and $t_y = 0$ is needed to bring each pair of subimages into perfect alignment. Unfortunately, the optimization scheme may fail to locate the matching parameters due to the interpolation artifacts. The second reason is associated with the content of the two images under analysis as well as the multiscale non-adaptive partition scheme. As the decomposition level increases, the two images are partitioned into subimages with smaller dimensions. For this specific pair of SAR images, a number of subimages become feature-less, a direct consequence of which is the flat mutual information that is less sensitive to transformations. Therefore, misregistration is more likely to occur for those subimages containing weak texture than those containing strong features.

4. CONCLUSIONS

In this paper we investigated the limitation of mutual information as a registration criterion for SAR images due to the inherent presence of speckle noise. To make this similarity function applicable for SAR image registration, we proposed two preprocessing steps. The first is to perform speckle reduction prior to registration, and the second is to determine an appropriate number of bins when estimating the joint histogram between the images to be registered. Experimental results indicate that they significantly reduce the interpolation artifacts present in the mutual information function; as a consequence, they allow for accurate registration. Equipped with the above two steps, a semi-automatic multiresolution SAR registration framework was devised where the optimal spatial alignment is achieved through progressive quadruple division, local rigid transformation and global TPS based elastic transformation. An example of registering a pair of JERS-1 and RADARSAT images proved the potential of this algorithm.

Figure 3: JERS-1 and RADARSAT SAR image registration

5. REFERENCES