A Novel Approach to SVD-based Image Filtering Improvement

Du xiaofeng
Department of Automation
Xiamen University
Xiamen, China
Li cuihua
Department of Computer Science
Xiamen University
Xiamen, China
Yang dunxu
Department of Computer Science
Xiamen University
Xiamen, China
Li jing
School of Software Engineering
BUPT
Beijing, China

Abstract—Image filtering based on SVD favors the denoising in the line (horizontal) and column (vertical) direction. Based on this property, a novel approach to improving the filtering efficiency of a noisy image is proposed in this paper. The new denoising method adapts shape and size of block to local orientation before performing SVD filtering. Through over-complete representation in overlap regions; the proposed method performs well in denoising and preserving image details. This new technique makes a contrast with some published denoising algorithms.

Keywords: SVD, Filter, Adaptive, Orientation Detection

I. INTRODUCTION

Image denoising is one of important issues in image processing. Traditionally, images are assumed to be linear-shift invariant and linear methods such as mean filter, Wiener filter [1], which are employed to solve the problem. Although these approaches are simple and cheap to implement, they fail to preserve details (e.g. texture, edge) of image. To better preserve image details, many researchers paid their attention to transform-domain, and nonlinear methods which have become the mainstream.

The Singular Value Decomposition (SVD) is a classical tool used in two-dimensional grey scale image processing [2]. An image may be decomposed into a sum of rank one matrices (images) that can be termed as eigenimages. The eigenvectors (eigenimages) form an orthogonal basis for representation of individual images in the image set. Under the assumption of additive noise, the original image is usually estimated to a lower rank approximation of the noisy image. This model implies that the largest eigenvalues are associated to the signal components and the lowest eigenvalues to the noise components. The threshold selection has been extended in several ways. In [3], the author suggests performing SVD filtering in specific sub-bands of wavelet domain by adaptively selecting threshold based on the in-homogeneous nature of image, [4] and tries to find the relation between threshold and the sub-bands of image that is decorrelated by IntDCT.

Recently, Many denoising algorithms [5-6] argue that an over-complete representation of the signal is superior to image denoising. The main advantage of over-complete expansion mainly lies in suppression of the Gibbs phenomena. The translation invariant denoising algorithm is achieved by shifting the signal multiple times, denoising each shifted signal separately (using orthogonal de-composition for each shift), shifting back and then averaging the results. In the process of denoising shift versions of the signal, edge artifacts occur at different locations. Where the signals are shifted back and averaged these edge artifacts are averaged as well. In [6], the author demonstrated this phenomenon using the sliding window transform-domain image denoising methods. The overlap between successive windows accounts for the over-completeness, while the transform itself is typically orthogonal (e.g. the SVD).

In this paper, we propose a novel method called “Orientation Adaptive SVD (OA-SVD)” to improve SVD-based image filtering. The algorithm adaptively changes filtering regions according to their local orientation feature before performing SVD filter. Thanks to the overlaps, we avoid blocking artifacts and further improve the denoising efficiency. The paper is organized as follows. A short review of SVD is provided in Section 2. Section 3 is about our

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orientation adaptive SVD. Experimental results and discussion are presented in Section 4. Finally, conclusion is given in Section 5.

II. SVD REVIEW

Any image of size \( M \times N \) \((N \geq M)\) can be treated as real-valued matrix \( X \) \([2]\). In practice, in an additive noise model, we observe a matrix \( A = X + E \), where \( E \) is a random noise perturbation matrix of full rank. Matrix \( A \) can be decomposed uniquely as

\[
A = USV^T = \sum_{i=1}^{R} \lambda_i A_i
\]

(1)

Where \( U \) and \( V \) are orthogonal matrices and \( S = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_R) \) is a diagonal matrix. The diagonal elements of \( S \) can be arranged in a descending order and are called the singular values of \( A \). Suppose that the first \( R \) singular values of \( A \) are introduced by a true signal, then the last \( N - R \) singular values caused by noise can be small (but not necessarily zero). By setting zero to the “non-significant” singular values, we can estimate the true signal.

III. ORIENTATION ADAPTIVE SVD

In \([7]\), the authors give a deeper analysis of the SVD filter and stress on the equivalence between a matrix SVD and simultaneous Principal Component Analysis (PCA) of line vectors and column vectors. They also point out that the left (resp. right) singular vectors of matrix \( A \), which are matrix \( AA^T \) (resp. \( A^T A \)) eigenvectors, are the principal components of the column (resp. line) vector set. Since that image filtering based on SVD favors the denoising in the line (horizontal) and column (vertical) direction, they propose a simple algorithm based on PCA-filtering processed on a rotated image so that the principal directions become vertical or horizontal. Intuitively, we can use this property to improve the filtering efficiency by skewing the block shape so that the main orientation of block becomes parallel to lines (or equivalently to the columns), after performing the classical SVD, the signal information will be in the smaller number of important components, and the fewer components to describe the signal, the better will the restored image quality be.

In the following sections, we denote the current block as \( A_i \) (the size of \( A_i \) is \( N \times N \)), the region to be SVD is termed \( C_i \).

A. Orientation detection

To get the orientation of \( A_i \), we adopt the robust orientation method proposed by X. Feng, and the relevant details can be found in \([8]\).

For each block \( A_i \), we group its gradient maps \((g_x, g_y)\) into an \( N \times 2 \) matrix. And then compute the truncated SVD of matrix \( G_i \), \( G_i = U_i S_i V_i^T \), \( S_i \) is a diagonal \( 2 \times 2 \) matrix representing the energy in the dominant directions:

\[
\begin{bmatrix}
s_{11} & 0 \\
0 & s_{22}
\end{bmatrix}
\]

The second column of the \( V_{12} = [v_{11}, v_{12}]^T \), defines the dominant orientation angle.

Here, we define two parameters:

\[
\lambda_i = \frac{s_{11} - s_{22}}{s_{11} + s_{22}},
\]

(2)

\[
\gamma_i = \frac{s_{11}s_{22}}{N}
\]

1. If \( s_{11} = s_{22} = 0 \), \( \gamma_i \) is close to zero, and the block is quite smooth, and SVD can be performed directly on original region, namely, \( C_i = A_i \), and the threshold is set to \( T_{\text{smooth}} \).

2. If \( s_{11} \gg s_{22} \), \( \lambda_i \) is close to 1, and the block has significant orientation, and its orientation can be calculated:

\[
\alpha_i = \arctan \left( \frac{v_{11}}{v_{12}} \right)
\]

(4)

SVD is performed after transformation of the block as is mentioned in 3.2, and the threshold is equal to \( T_{\text{smooth}} \).

3. If both \( s_{11} \) and \( s_{22} \) is high, \( \gamma_i \) is also large, the block includes high frequency signal or multiple orientations (e.g. texture or corner). We set \( C_i = A_i \). But in such case, its energy is spread over many components after SVD, so the threshold is smaller than \( T_{\text{smooth}} \):

\[
T_{HF} = \beta T_{\text{smooth}}
\]

(5)

Where \( \beta < 1 \).

B. Orientation Adaption

Suppose that there is an edge in block \( A_i \) and the angle between the edge and the vertical direction is \( \alpha_i \) \((0^\circ \leq \alpha_i \leq 45^\circ)\), so the range of \( \tan \alpha_i \) is: \([0, 1]\). One simple means to make the principal directions of an image vertical is to elongate the region size of \( A_i \) and skewing. Figure 1 is the illustration of this transformation. Region \( A_i \) (Figure 1. a) is expanded
to $C_j$ (Fig. 1. b) at first. $C_j$ is the smallest external parallelogram that includes $A_i$, and one of $C_j$ axis $y_2$ is parallel to edge. At last, the edge becomes vertical after skewing $C_j$ as Fig. 1. c.

Figure 1. Region transformation

For a digital image, we can implement the above transformation by resampling. In discrete space which involves only shifting each row of the image region $C_j$ to the left by $(i−1)\times gα_i−\lfloor gα_i\rfloor$ pixels, where $i$ is the row number (from top to bottom) and $\lfloor \cdot \rfloor$ denotes rounding to the nearest integer. For each row, $(i−1)\times gα_i−\lfloor gα_i\rfloor$ pixels are sampled. Fig. 2 is explanation for resampling. The original block has a diagonal line, so its orientation is 45°. After resampling, the diagonal edge becomes vertical.

Figure 2. Resampling

If the orientation angle of block $α_i$ is between $(45°,180°]$ , matrix transposition of the image blocks’ rows and columns should be employed first to make sure that the range of tangent of $α_i$ is: $[0,1]$. In this way, the orientation becomes aligned vertically (horizontally) the most favorable orientations for SVD filter.

C. **Denoising algorithm**

We use the divide-and-conquer techniques for image denoising.

1. Partition the image into overlapping blocks.
2. For every block $A_i$, estimate $\hat{λ}_i$ and $γ_i$.
3. If $\hat{λ}_i \geq 0.9$, then goto 4, else goto 5.
4. Calculate the orientation of $A_i$, and apply adaption mentioned in 3.2. Set the threshold as $T_{smooth}$, then goto 7.
5. If $γ_i < 50$, $A_i$ is quite smooth, so set $C_i = A_i$, and the threshold is set to $T_{smooth}$, then goto 7, else goto 6.
6. $A_i$ has high-frequency signal, set threshold as $T_{hf} = βT_{smooth}$, $β = 1/4$, and $C_i = A_i$, goto 7.
7. Perform classical SVD filter for $C_i$.

After processing all blocks, the final estimate is the weighted average of all overlapping local block-estimates. The weight is defined as:

$$\omega_i = \frac{1}{\text{num}_{\text{nonzero}} + 0.1}$$

where $\text{num}_{\text{nonzero}}$ is the number of nonzero singular values. Namely, the smaller the number of nonzero singular values, the smaller will the weight be. In our algorithm, parts of overlaps are between successive blocks, while parts are produced by elongation of blocks (Fig. 1. b).

IV. **EXPERIMENTAL RESULTS AND DISCUSSION**

To evaluate the performance quantitatively, two commonly used measures, the mean-square error (MSE) and the improvement of signal - noise ratio (ISNR), are computed:

$$\text{MSE} = \frac{1}{mn} \| f_o - f_f \|_2^2$$

$$\text{ISNR} = 20 \log_{10} \left( \frac{\| f_o - f_f \|_2}{\| f_o - f_f \|_2} \right)$$

Where $f_o, f_f, f_b$ are the clean image, the image after filtering and the image with noise respectively, and $\| \cdot \|_2$ represents $L^2$ norm.

All images are of size $512 \times 512$ and 8-bit per pixel. Each image was corrupted by additive i.i.d. Gaussian noise $N(0, σ^2)$ , and $σ = 10$. In our experiment, the original block size is $8 \times 8$, and the threshold $T_{smooth} = 60$. Table 1 summarizes the MSE of restored images by four denoising methods. ISNR is shown in Table 2. Data signed by “*” come from [3]. For highly textured images like the Barbara the OA-SVD can achieve satisfactory performance, and it performs better on other test images than the rest three methods not only on objective criteria but on visual fidelity as well, see Fig. 4 and Fig. 5.
Table 1. MSE of restored images

<table>
<thead>
<tr>
<th></th>
<th>BSVD*</th>
<th>Bayes Shrink*</th>
<th>WASVD*</th>
<th>OA-SVD</th>
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<tbody>
<tr>
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<td>Pepper</td>
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Table 2. ISNR of restored images

<table>
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<tr>
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<th>Bayes Shrink*</th>
<th>WASVD*</th>
<th>OA-SVD</th>
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</table>

V. CONCLUSION

This paper presents an image denoising approach by performing orientation adaptive SVD. Through adaptively changing the size of filtering regions and further estimating over-complete representation of overlap regions; the SVD filtering performance is further improved. There is another block-based filter (e.g. DCT, FFT) also favoring the denoising in the line (horizontal) and column (vertical) direction, which can also be improved by orientation adaption. However, the performance of orientation detection must be robust to noise.

REFERENCES

