Dynamic analysis of steel frames with flexible connections

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Received 1 June 2001; accepted 14 February 2002

Abstract

This paper deals with the effects of flexibility and damping in the nodal connections on the dynamic behavior of plane steel frames. A flexible eccentric connection is idealized by nonlinear rotational spring and dashpot in parallel. Thus, the effects of viscous and hysteretic damping on dynamic response of frame structures are taken into consideration. A numerical model that includes both nonlinear connection behavior and geometric nonlinearity of the structure is developed. The complex dynamic stiffness matrix for the beam with flexible connections and linear viscous dampers at its ends is obtained. Several examples are included to illustrate the efficiency and accuracy of the present model. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Steel frame; Semi-rigid connection; Nonlinear dynamic analysis

1. Introduction

The conventional methods of analysis and design of frame structures are based on the assumption that the joint connections are either fully rigid or ideally pinned. The models with ideal connections simplify analysis procedure, but often cannot represent real structural behavior. Therefore, this idealization is not adequate as all types of connections are more or less, flexible or semi-rigid. It is proved by numerous experimental investigations that have been carried out in the past [1–4].

Based on experimental study due to static monotonic loading tests for various types of connections, many models have been done to approximate the connection behavior. The simplest one is the linear model that has been widely used for its simplicity [5–7]. However, this model is good only for the low level loads, when the connection moment is quite small. In each other case, when the connection rigidity may rapidly decrease compared with its initial value, a nonlinear model is necessary. Several mathematical models to describe the nonlinear behavior of connections have been formulated and broadly used in research practice [8–11].

So far, most experimental and theoretical work is limited to static analysis of steel frames with flexible connections. Very few papers have been devoted to the dynamic analysis although the flexibility of connections with energy dissipation has a great influence on dynamic behavior of these types of structures. Under cyclic loads, the connection hysteresis loop increases the energy absorption capacity and hysteretic damping may significantly reduce dynamic response of real structures. Therefore, modelling of the nodal connection is important for the design and accuracy in the dynamic frame structure analysis. However, as the experimental data for the connection behavior under cyclic loading are rather poor, it is difficult to make corresponding mathematical model. The experiments carried out by Popov and coworkers [12–16] and also in Refs. [17,18] show that the hysteresis loops under repeated and reversed loading are very stable, so the moment–rotation functions obtained by static tests can be extended to the dynamic analysis.

The dynamic analysis of frames with flexible connections using linear moment–rotation relationship has been studied in several papers [19–21]. Kawashima and Fujimoto [20] obtained the complex dynamic stiffness

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matrix for a uniform beam element with linear rotational springs and dashpots at its ends. The influence of the flexibility and eccentricity in the connections on dynamic behavior of plane frames within the linear theory was investigated by Suarez et al. [21]. Although the linear constitutive model of nodal connections is very easy to use, it is inadequate in the term that it is applicable only to a small range of the initial rotations and because it cannot represent hysteretic damping to be a primary source of passive damping in the frame structures. A bilinear moment–rotation function, which is also accurate only for a small rotation range, was used by Sivekumaran [22] and Yousef-Agha et al. [23]. The effect of hysteretic damping resulting from the nonlinear flexible connection on the dynamic response of the frame was studied by Shi and Altury [24]. They developed a numerical model based on the complementary energy approach using the Ang and Morise [6] function for the moment–rotation relations at connections. Chan and Ho [25] proposed a numerical method for linear and nonlinear vibration analysis of frame with semi-rigid connections. They adapted the conventional cubic Hermitian shape functions for a uniform beam with end springs and derived the element matrices using the principle of total potential energy. The influence of both hysteretic and viscous damping at connections on seismic response of the steel frames was considered by Sekulovic et al. [26]. The combined effects of material yielding and connection flexibility in static and dynamic problems have been discussed in detail by Chan and Chui [36]. Several nonlinear flexible connection models under cyclic loading were established in the past decade [27–30].

The present study is an extension of the author’s previous work [31], regarding static analysis of flexible joint frames. A more general case of the dynamic analysis. Two types of nonlinearities are considered: geometric nonlinearity of the structure and material (constitutive) nonlinearity of the connections. These nonlinearities are interactive. The eccentricity of the connections is also considered. To describe the nonlinear behavior of the connection under cyclic loading, the independent hardening model is used. So, the effect of hysteretic damping on dynamic behavior of the structure is directly included through the connection constitutive relation. Moreover, the influence of viscous damping at connections on dynamic response of frame structures is considered. For a uniform beam with rotational springs and dashpots attached at its ends the complex dynamic stiffness matrix is obtained. The stiffness matrix has been obtained based on analytical solutions of governing differential equations second order analysis, so that each beam represents one element. Nodal displacements and rotations are chosen as the primary unknowns, while displacements and rotations of the element ends are eliminated. Thus, the number of degrees of freedom are the same as for the system with rigid connections. Besides, the consistent mass and damping matrices are derived. These matrices are based on the physical properties of the member and given in an explicit form. The present matrices are more general than the corresponding matrices previously obtained by other authors.

Energy dissipation exists in frame structures under dynamic loads. The primary sources of energy dissipation, is known, may be hysteretic behavior of connections and the friction between elements forming the beam–column assemblage. In addition, different types of energy dissipation devices can be installed into connections in order to increase the structural energy absorption capacity. For this reason, in the present model, the total energy dissipation is confined to the joint connections. Two types of energy dissipation are assumed. They are: hysteretic damping due to nonlinear behavior of connections and viscous damping at the connections. In general, the effects of these dampings are coupled. Also, they can be considered separately using either linear constitutive relation for the connections or zero value for the viscous damping coefficients at the connections. As it is assumed that all structural elements, except the connections, remain elastic through the whole loading range, the energy dissipation at plastic hinges cannot be observed. Also, energy dissipation due to radiation damping at the supports is not included in this consideration. The other types of energy dissipation that may exist in real frame structures can be included in the present model in the usual way, by mass and stiffness proportional damping matrix.

Based on theoretical problem formulation, a computer program was developed in order to increase dynamic analysis efficiency and design of steel frames. A parametric study has been performed in order to estimate the influence of nonlinear connection flexibility and viscous damping at connections on the frame dynamic (seismic) response. The present numerical model is restricted to 2D frame systems. It can be extended on a more general case of 3D analysis without difficulty. Besides, the proposed beam element can be easily incorporated into existing commercial programs for structural analysis.

2. Formulation of structural element

A beam element with flexible, eccentric and viscous damping connections is shown in Fig. 1. The flexible connections are represented by nonlinear rotational springs at beam ends. Thus, only the influence of bending moment on the connection deformation is considered, while the influences of axial and shear forces are neglected. The connection spring element is assumed massless and dimensionless in size. The eccentricity is
modelled by short infinitely stiff elements whose lengths are \( e_1 \) and \( e_2 \). The linear viscous damping at nodal connections are modelled by dashpots acting at beam ends.

2.1. Stiffness matrix and nodal force vector

The stiffness matrix and the nodal force vector for the flexible eccentric beam have been represented in a previous work by the authors [31]. It will be briefly summarized herein. Joint displacements and rotations are the primary unknowns, while displacements and rotations of the beam ends can be eliminated as has been shown in [31]. Thus, the number of degrees of freedom is the same as for the beam element with fully rigid connections. Consequently, the function describing lateral displacement \( v(x) \) for the element with flexible eccentric connections can be written in the usual way by interpolation function matrix and nodal displacements vector as:

\[
v(x) = \mathbf{N}(x)(I + \mathbf{G})\mathbf{q} = \tilde{\mathbf{N}}(x)\mathbf{q},
\]

where

\[
\mathbf{N}(x) = [N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x)],
\]

\[
\mathbf{q}^T = \{v_1 \quad \phi_1 \quad v_2 \quad \phi_2\},
\]

\[
\mathbf{G} = \frac{1}{A} \begin{bmatrix} 0 & \Delta e_1 & 0 & 0 \\ g_{21} & g_{22} & g_{23} & g_{24} \\ 0 & 0 & 0 & -\Delta e_2 \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix},
\]

denoting the matrix of interpolation functions obtained based on the analytical solutions of the second order analysis equations [32], the nodal displacement vector and the correction matrix, respectively. Interpolation functions \( N_i(x) \), \( i = 1, \ldots, 4 \) and elements of correction matrix \( \mathbf{G} \) are given in Appendix A.

Stiffness matrix for the beam element with flexible eccentric connection is obtained through the total potential energy, that can be written as

\[
U = U_a + U_i + U_s,
\]

where

\[
U_a = \frac{EI^2}{2A} k^4 l,
\]

\[
U_i = \frac{EI}{2} \int_0^l v_{x,1}^2 \, dx,
\]

\[
U_s = \sum_{i=1}^2 c_i x_i^2,
\]

denoting strain energy of the beam, axial (\( U_a \)) and flexural (\( U_i \)) and strain energy of the springs (\( U_s \)). Strain energy due to axial deformation and bending are coupled since parameter \( k^2 \) includes derivatives of both axial and lateral displacements [31]. With the assumption that \( k^2 = \text{const.} \), these two part of the strain energy can be expressed independently. Thus, after substituting Eq. (1) into Eq. (4b) the following can be obtained:

\[
U_i = \frac{1}{2} \mathbf{q}^T (\mathbf{k}_{ii} + \mathbf{k}_{ef}) \mathbf{q},
\]

where matrices \( \mathbf{k}_{ii} \) and \( \mathbf{k}_{ef} \) are defined as

\[
\mathbf{k}_{ii} = EI \int_0^l \left[ (\mathbf{N}'(x))^T \mathbf{N}'(x) \right] \, dx,
\]

\[
\mathbf{k}_{ef} = \mathbf{G}^T \mathbf{k}_{ii} + \mathbf{k}_{ii} \mathbf{G} + \mathbf{G}^T \mathbf{k}_{ii} \mathbf{G},
\]

denoting beam stiffness matrix with the rigid connections according to the second order analysis and correction matrix that accounts for the effects of flexibility and eccentricity, respectively. Analytical expression for the elements of matrix \( \mathbf{k}_{ii} \) and the appropriate expansions in the power series form, convenient for the numerical analysis, are given by Goto and Chen [33].

The simplified form of this matrix, corresponding to the linearized second order analysis, can also be used. In that case, the stiffness matrix has the form

\[
\mathbf{k}_{ii} = \mathbf{k}_0 + \mathbf{k}_{ef},
\]
where $k_c$ is the conventional stiffness matrix and $k_e$ is the geometric stiffness matrix of the uniform beam. In this case, the simplified form of the matrix $G$, with functions $\phi_i, i = 1, \ldots, 4$, are replaced by 1.0, can be used.

The strain energy of the springs can be expressed in the form

$$U_s = \frac{1}{2} q^T k_s q.$$  

(8)

where

$$k_s = \mathbf{G}^T C \mathbf{G}.$$  

(9)

The explicit form of matrices $\mathbf{G}$ and $C$ can be found in [31]. From Eqs. (5) and (8) the total strain energy due to the bending for the beam with flexible and eccentric connections can now be written as:

$$U = U_t + U_s = \frac{1}{2} q^T (k_{ii} + k_{ef} + k_e) q.$$  

(10)

The equivalent generalized end force vector due to distributed loads along the beam $p(x)$ is obtained in the usual manner:

$$Q = \int_0^l p(x) \mathbf{N}^T(x) \, dx = (\mathbf{I} + \mathbf{G})^T \int_0^l p(x) \mathbf{N}^T(x) \, dx.$$  

(11)

Components of the vector $Q$, for some simple load distributions and temperature change are given in the closed form in [32]. In general case, elements of the vector $Q$ are computed numerically.

2.2. Element mass matrix

Assuming that the mass density $\rho$ is constant, the element consistent mass matrix $m$ can be formulated as:

$$m = \int_0^l \rho \tilde{\mathbf{N}}^T(x) \tilde{\mathbf{N}}(x) \, dx,$$  

(12)

where $\tilde{\mathbf{N}}(x)$ is the matrix of modified shape functions defined by Eq. (1). After substitution of Eq. (1) into Eq. (12), the consistent element mass matrix, for the uniform beam with flexible eccentric connections, can be written as:

$$m = m_0 + m_{ef},$$  

(13)

where

$$m_0 = \int_0^l \rho \mathbf{N}^T(x) \mathbf{N}(x) \, dx,$$  

(14a)

$$m_{ef} = \mathbf{G}^T m_0 + m_0 \mathbf{G} + \mathbf{G}^T m_0 \mathbf{G}.$$  

(14b)

In the above relations, $m_0$ denotes conventional mass matrix for the beam element and $m_{ef}$ denotes the mass correction matrix.

2.3. Complex dynamic stiffness matrix

Apart from nonlinear rotational springs, rotational viscous dashpots are attached at beam ends, as shown in Fig. 1. The total moment at each nodal connection ($i = 1, 2$) can be given in terms of relative rotation $\theta$ between beam end and column face and relative angular velocity $\dot{\theta}(t)$ as:

$$M_i(t) = k_i \theta_i(t) + c_i \dot{\theta}_i(t), \quad i = 1, 2.'$$  

(15)

where $k_i$ and $c_i$ are rotational spring stiffness and rotational viscous damping coefficients, while dot over the symbols denotes differentiation with respect to time. The tangent or secant form of the above relation may be written if nonlinear springs and/or dashpots are considered. In the case of periodic response with circular frequency $\omega$ the following relation between the amplitudes may be derived:

$$M_{i(0)} = k_i \theta_{i(0)}, \quad i = 1, 2',$$  

(16)

where complex flexural stiffness $k_i^*$ of the connection is defined as the ratio between moment and relative rotation amplitudes:

$$k_i^* = \frac{M_i(t)}{\theta_{i(t)}}, \quad \dot{\theta}_{i(t)} = \theta_{i(0)} e^{i\omega t}.$$  

(17a)

(17b)

The beam end force vector $\mathbf{R}(t)$ can be expressed in terms of the end displacement vector $\mathbf{q}(t)$ and relative end rotation vector $\theta(t)$ as:

$$\mathbf{R}(t) = k \{ \mathbf{q}(t) - \theta(t) \},$$  

(17a)

where

$$\mathbf{R}(t) = \{ \mathbf{T}_1(t) \quad \mathbf{M}_1(t) \quad \mathbf{T}_2(t) \quad \mathbf{M}_2(t) \},$$  

(18)

$$\mathbf{q}(t) = \{ v_1(t) \quad \phi_1(t) \quad v_2(t) \quad \phi_2(t) \},$$  

(19)

$$\theta(t) = \theta_{1(0)} e^{i\omega t},$$  

(20)

are end force vector, end displacement vector and relative end rotation vector of the member respectively (Fig. 1), while $k$ is the classical or the second order flexural stiffness matrix of a uniform beam, that depends on the type of analysis.

After the elimination of relative end rotation vector $\theta(t)$, Eq. (17a) transforms to:

$$\mathbf{R}(t) = k^* \mathbf{q}(t),$$  

(20)

where

$$k^* = (\mathbf{I} - \mathbf{S})^T \mathbf{k} (\mathbf{I} - \mathbf{S}) + \mathbf{S}^T \mathbf{k} \mathbf{S} = \mathbf{k} + \mathbf{k}_1 + \mathbf{k}_2,$$  

(21a)

$$\mathbf{k}_1 = -\mathbf{k} \mathbf{S} - \mathbf{S}^T \mathbf{k},$$  

(21b)
\( k_2 = S^T (k + K) S \),

\( \dot{q}(t) = q(0) e^{j\omega t}. \)  

The explicit forms of matrices \( k_1 \) and \( k_2 \) are given in Appendix A. The matrix \( S \) can be obtained from matrix \( G \) putting \( e_1 = e_2 = 0 \), and it can be found in Appendix A. The matrix \( k' \) is a complex flexural stiffness matrix of uniform beam with flexible connection according to the linear or second order analysis, including both flexible and viscous phenomena. The elements of this matrix corresponding to the linear analysis are given in Appendix A.

Expanding the elements of the dynamic stiffness matrix in series with respect to the circular frequency \( \omega \) and neglecting higher terms than the third order, the following expansion is obtained in the decomposed form:

\[
\begin{align*}
\dot{k}' &= k + j \omega c - \omega^2 m. 
\end{align*}
\]

where \( k \) is the static stiffness matrix; \( c \), the damping matrix and \( m \), the mass matrix for the uniform beam with flexible springs and dashpots at its ends.

It should be noted that if neither eccentricities nor springs and dashpots are present, the matrix \( k \) transforms to the classical beam element flexural stiffness matrix \( k_0 \). The matrix \( c \) is consistent element damping matrix, which is based on physical properties of the member. The elements of the matrix \( c \) and matrix \( m \) for a uniform beam according to the linear analysis are provided in Appendix A.

The proposed viscous damping at beam ends causes that viscously damped system does not satisfy Caughey and O’Kelly’s condition [34]. The response of a multi-degree-of-freedom system cannot be expressed as a linear combination of its corresponding modal responses. So, the system is nonclassically damped and it generally has complex valued natural modes. It is necessary to elucidate physical interpretation of solutions represented by complex conjugate pairs of characteristic values. In order to establish the relationship between coefficient \( c_i \) of viscous damping in joints and modal relative damping factor \( \xi_i \) for \( k \) mode shape, a specific procedure has been established.

Provided that the amount of damping in the system is not very high, the characteristic values occur in complex conjugate pairs with either negative or zero real parts. Let \( \lambda_i \) and \( \bar{\lambda}_i \) be a pair of characteristic values defined by:

\[
\begin{align*}
\lambda_i &= -\bar{\epsilon}_i + j \omega \omega_i, \\
\bar{\lambda}_i &= -\bar{\epsilon}_i - j \omega \omega_i.
\end{align*}
\]

Further, let \( \omega \omega_i \) be the modulus of \( \lambda_i \), i.e.:

\[
\omega \omega_i = \sqrt{\epsilon_i^2 + \omega_0^2}.
\]

The corresponding pseudo-damping factor \( \zeta_i \) is:

\[
\zeta_i = \frac{\epsilon_i}{\omega \omega_i}.
\]

Based on the parametric study, the relationship between coefficient \( c_i \) of viscous damping in joints and modal pseudo-relative damping factor \( \zeta_i \) for \( i \) mode shape can be obtained, and the corresponding curve is presented in Fig. 2.

3. Semi-rigid connection modelling

Numerous experimental results have shown that the connection moment–rotation relationships are nonlinear over the entire range of loading for almost all types of connections. To describe connection behavior, different mathematical models have been proposed. In this study, the three parameter power model proposed by Richard and Abbott [8] and Kishi et al. [27] is used to represent moment–rotation behavior of the connection under monotonic loading. This model can be formulated as:

\[
M = \frac{k_0 \theta}{[1 + (\theta/\theta_0)^n]^{1/n}},
\]

where \( k_0 \) initial connection stiffness; \( n \), the shape parameter; \( \theta_0 = M_u/k_0 \), the reference plastic rotation and \( M_u \), the ultimate moment capacity. Accordingly to Eq. (26), \( M-\theta \) functions for the two types of connection (double web angle (DWA), top and seat angle with double web angle (TSDWA)) are shown in Fig. 3a. The first of these connections are rather weak and the second is relatively stiff. The details of these connections can be found in Ref. [35].

The independent hardening model was adopted to simulate the inelastic connection behavior under cyclic loading. In this model, the characteristics of connections are assumed to be unchanged through the loading.
cycles. The moment–rotation curve under the first cycle of loading unloading and reverse loading remain unchanged under the repetition of loading cycles. The skeleton curve used in the model was obtained from three parameter power model. The cyclic moment–rotation curve based on this model is schematically shown in Fig. 3b. The independent hardening model is simple and easily applicable to all types of steel frames connection models. More information about this model can be found in Refs. [35,36].

4. Numerical procedures

The equations of motion of a frame subjected to dynamic loading can be written in the following form:

\[ \mathbf{\ddot{M}} \mathbf{U} + \mathbf{\dot{C}} \mathbf{U} + \mathbf{K} \mathbf{U} = \mathbf{F}, \]  

(27)

or

\[ \mathbf{M} \mathbf{\ddot{U}} + \mathbf{C} \mathbf{\dot{U}} + \mathbf{K} \mathbf{U} = -\mathbf{M} \mathbf{\ddot{U}}_g, \]  

(28)

in which \( \mathbf{M} \) is the mass matrix, \( \mathbf{C} \) is viscous damping matrix and \( \mathbf{K} \) is static stiffness matrix for the system of structural elements. The time dependent vectors \( \mathbf{U}, \mathbf{\dot{U}} \) and \( \mathbf{U}_g \) are the relative node accelerations, velocities and displacements respectively, while the vectors \( \mathbf{F} \) and \( \mathbf{U}_g \) are externally applied loads and ground accelerations. The equations of motions are integrated using step-by-step integration, with a constant acceleration assumption within any step.

To solve the nonlinear equations, that are nonlinear in terms of the displacements as well as the axial force, secant stiffness method is used. This method is very simple in computer implementation and also gives convergent solutions for design loadings. In each time step, the load increment \( \Delta \mathbf{F} \) or \( \Delta \mathbf{U}_g \) is divided into a few smaller subincrements (Fig. 4) and iterative procedures are employed. The iterative algorithm is based on evaluating secant stiffness matrix, which depends on the stiffness of connections, represented by slope of its moment–rotation curve at any particular moment value. The convergence is obtained when the differences be-

![Fig. 3. (a) Three parameter power model and (b) independent hardening model.](image)

![Fig. 4. Secant stiffness method in a case of nonlinear connection behavior.](image)
tween two consecutive cycles displacements at all joints reach the prescribed tolerance. The current connection stiffness becomes the starting connection stiffness for the next load subincrement. The convergent solutions for all load subincrements are accumulated to obtain the total nonlinear response within time step.

5. Numerical examples

Based on the above theoretical considerations, a computer program has been developed and dynamic analysis of plane frames with different number of stories and bays, as well as different types of connections and loads, has been performed. For illustration, only some typical results are presented herein.

5.1. Ten-storey single bay frame

Ten-storey single bay plane steel frame 40.00 m high and 8.00 m wide has been analyzed. The geometrical and material properties of this frame are shown in Fig. 5. Two types of semi-rigid beam-to-column connections (TSDWA and DWA) with both linear and nonlinear moment–rotation relations were considered. For comparison the same frame with rigid joints was analyzed. Linear (first order) and a geometrically nonlinear (second order) analyses of the frame were carried out for all aforementioned connection types. The results of linear and nonlinear analyses of the frame with fully rigid joints obtained by this study have been compared with the corresponding results obtained by the known software package SAP 2000 [37]. These two sets of results are quite close to each other. For the semi-rigid type connections database developed by Chen and Kishi [38] was used. The following examples include vibration and transient analysis of steel frame shown in Fig. 5, if subjected to uniform and seismic ground excitations.

5.1.1. Natural frequencies

The natural frequencies and the corresponding periods for the first three modes are determined for the cases of fully rigid and linear semi-rigid connection (DWA, TSDWA) and shown in Table 1. The change in natural frequencies due to variation of joint stiffness (fixity factor) is shown in Fig. 6. The natural frequencies are normalized by dividing their values by the frequencies obtained for the frame with rigid joints and fixity factors are defined as:

\[ k_{0} = 3EI \left( \frac{\gamma_{i}}{1-\gamma_{i}} \right) \]

where \( k_{0} \) is initial connection stiffness that varies from 0 in the case of pinned connection to \( \infty \) for the case of fully rigid connection and \( \gamma_{i} \) is fixity factor whose values are normalized from 0 to 1.

As seen in Fig. 6, the connection flexibility has a significant influence on variation of the natural frequencies particularly on the lower frequencies. This fact can be

<table>
<thead>
<tr>
<th>Type of connection</th>
<th>Natural frequencies (rad/s)</th>
<th>Periods (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First mode</td>
<td>Second mode</td>
</tr>
<tr>
<td>Rigid</td>
<td>6.328</td>
<td>17.523</td>
</tr>
<tr>
<td>TSDWA</td>
<td>5.727</td>
<td>16.088</td>
</tr>
<tr>
<td>DWA</td>
<td>4.647</td>
<td>13.519</td>
</tr>
</tbody>
</table>

Fig. 5. Layout and properties of single-bay ten-storey frame investigated.
very important for seismic analysis of frame structures, as the lower modes may generally have the principal influence on seismic response of buildings.

Fig. 7 shows the influence of eccentricity beam-to-column connection on the variation of natural frequencies. It can be seen that the eccentricity of connections may have a practical influence depending on the type and size of the connection.

5.1.2. Transient analysis

The transient displacement analysis of the frame shown in Fig. 5 has been performed for the two cases of ground motions: two steps sudden acceleration and an earthquake excitation. Gravitational loads are also included and they are considered as additional lumped masses at the beam nodes.

5.1.3. Two-steps ground acceleration

The frame is assumed to be subjected to the sudden discontinuous two-steps uniform ground acceleration shown in Fig. 10. The transient response analysis of the frame with various connection types according to the first order and second order analyses has been carried out. Characteristic results of the lateral displacements and accelerations at the top of the frame as well as bending moments and shear forces at the base of the frame for the various types of connections are presented in Table 2. The effects of viscous damping at joint connections on the deflection and internal forces of the frame are also included. The envelopes of lateral displacement and shear force of the frame with various connection types according to the first order and second order analyses are plotted in Figs. 8 and 9.

It can be seen from Table 2 and Figs. 8 and 9 that the frame with semi-rigid connections has a larger lateral displacements, but smaller shear forces when compared with the fully rigid connection. These differences increase with decrease in the connection stiffness. Consequently, the difference in maximum displacement at the top of the frame with rigid joints and semi-rigid type of joints are (in percent): 15.1 and 48.6 for linear or 19.1 and 64.0 for nonlinear types of TSDWA and DWA connections, respectively. The differences (in percent) in the shear forces at the base of the frame are: 3.9 and 13.2 for linear or 5.5 and 20.9 for nonlinear types of TSDWA and DWA connections, respectively.

It is obvious that there is a significant difference between the results obtained for the frame with rigid joints and the frames with semi-rigid (DWA and TSDWA) connections especially in the case of the weak connections type (DWA).

The time histories of the lateral displacements at the left top node of the frame with various connection types according to the first order and second order analyses are plotted in Fig. 10. It can be seen from Fig. 10 that the frame with nonlinear connections has longer amplitudes and period when compared with the rigid joint case. The displacement amplitudes and period increase with a decrease in joint stiffness. They are longer in DWA connection case than in the TSDWA connection case. Besides, the nonlinear connections dampen and produced nonrecoverable deflection due to the presence of permanent deformations at connections. On the contrary, the frame with either fully rigid and linear
connection types produces no hysteretic damping. Fig. 10 shows also that the second order analysis further magnifies the aforementioned nonlinear effects on the frame deflection response.

Table 2
Maximum displacements and internal forces of frame investigated

<table>
<thead>
<tr>
<th>Type of connections</th>
<th>Maximum displacement of node A (cm)</th>
<th>Maximum bending moment of node B (kNm)</th>
<th>Maximum shear force of node B (kN)</th>
<th>Maximum acceleration of node A (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First order</td>
<td>Second order</td>
<td>First order</td>
<td>Second order</td>
</tr>
<tr>
<td>Rigid</td>
<td>Present study</td>
<td>2.84</td>
<td>3.10</td>
<td>161.16</td>
</tr>
<tr>
<td>SAP2000</td>
<td>2.85</td>
<td>3.10</td>
<td>161.50</td>
<td>166.60</td>
</tr>
<tr>
<td>TSDWA</td>
<td>Linear c = 0</td>
<td>3.27</td>
<td>3.58</td>
<td>164.33</td>
</tr>
<tr>
<td></td>
<td>Linear c = 50,000</td>
<td>2.96</td>
<td>3.24</td>
<td>154.49</td>
</tr>
<tr>
<td></td>
<td>Nonlinear c = 0</td>
<td>3.38</td>
<td>3.66</td>
<td>166.39</td>
</tr>
<tr>
<td></td>
<td>Nonlinear c = 50,000</td>
<td>3.00</td>
<td>3.30</td>
<td>153.66</td>
</tr>
<tr>
<td>DWA</td>
<td>Linear c = 0</td>
<td>4.22</td>
<td>4.60</td>
<td>178.02</td>
</tr>
<tr>
<td></td>
<td>Linear c = 50,000</td>
<td>2.96</td>
<td>3.46</td>
<td>137.93</td>
</tr>
<tr>
<td></td>
<td>Nonlinear c = 0</td>
<td>4.66</td>
<td>5.66</td>
<td>177.51</td>
</tr>
<tr>
<td></td>
<td>Nonlinear c = 50,000</td>
<td>3.68</td>
<td>5.36</td>
<td>150.98</td>
</tr>
</tbody>
</table>

Fig. 8. Lateral displacement envelopes (a) and shear force envelopes (b) according to the first order analysis.
Fig. 9. Lateral displacement envelopes (a) and shear force envelopes (b) according to the second order analysis.

Fig. 10. Time history displacement at the left top node of the frame with various connection types. According to the (a) first order analysis and (b) second order analysis.
The influence of viscous damping at connections on displacement response of frame with some types of connections is shown in Figs. 11 and 12. These figures show that viscous damping alters the deflection response of the frame significantly, particularly in case a weak connection type (DWA). It is obvious that the displacement response of the frame decays with time for both linear and nonlinear types of connections. In the linear type connection case plotted in Figs. 11a and 12a there is only viscous damping, so the frame oscillates about its initial position. In the nonlinear type connection case plotted in Figs. 11b and 12b, there are both viscous and hysteretic damping, so the frame oscillates about its permanent drift position which exists as a result of the permanent nonrecoverable rotations of connections.

5.1.4. Earthquake excitation

The frame is assumed to be subjected to the first four seconds of Montenegro earthquake (1979) NS component motion shown in Fig. 13b. The peak ground acceleration was 0.4 g at about third second. The displacement response at the top of the frame with two types of nonlinear connections and rigid jointed frame according linear and second order analyses plotted in Fig. 14. This figure shows considerable difference between the responses of rigid jointed frames and frames with nonlinear connection types. The main reason for it is hysteretic damping which exists only in nonlinear connection case. It reduced transient deflection response gradually decreasing its amplitude with time.

Fig. 14a shows that in both TSDWA and DWA nonlinear types of connection there are permanent deflection drift (due to large connection rotation) to the positive side (forward permanent deflection) in the first case, and to the negative side (backward permanent deflection) in the second case. Fig. 14b also shows that the frame with DWA connection demonstrates remarkably different response from the others. After about 3 s in this case, the frame deforms suddenly to a

![Fig. 11](image)

**Fig. 11.** The influence of viscous damping at connections on displacement response of the frame with TSDWA connections. (a) Linear type of connection and (b) nonlinear type of connection.
Fig. 12. The influence of viscous damping at connections on displacement response of the frame with DWA connections. (a) Linear type of connection and (b) nonlinear type of connection.

Fig. 13. Lateral load history. Accelerogram (a) and spectrum (b) for Montenegro earthquake (1979), Petrovac NS component.
peak value that reaches over 60 cm, and oscillates about this permanent deflection. The main reason for this is the appearance of large rotational deformations at the joint connections.

The lateral displacement and shear force envelopes of the frame with the previous connection types obtained according linear and second order analyses are shown in Figs. 15 and 16. The frames with flexible nonlinear connections under applied earthquake motion have smaller lateral displacements and shear forces when compared with the rigid jointed frame. It is necessary bear in mind that any earthquake is an excitation with a wide range of frequencies. The predominant frequencies of the applied earthquake are within the range from 2 to 10 Hz (periods 0.1–0.5 s). The lowest natural frequencies of the investigated frames (rigid, TSDWA, DWA) are much higher than the predominant earthquake frequencies, while the second and the third natural frequencies are within the range of predominant frequencies of the applied earthquake (Fig. 13b). It obviously has a great influence on displacement response of these frames.

Time history acceleration responses of the frame with rigid and two types nonlinear (TSDWA, DWA) connections according to linear and second order analyses are shown in Fig. 17. It is obvious that there is a substantial hysteretic damping effect on the acceleration response of the frame with nonlinear connections. On the contrary, in the case of rigid jointed frame, the acceleration response is not dampened, so the large amplification of the acceleration response exists. For the applied ground motion, the acceleration is amplified from the base to top of the frame by factors 6.5, 3.9 and 1.7 for rigid, TSDWA and DWA case of connections, respectively.

The hysteresis $M-\theta$ loops at joint C of the frame with TSDWA and DWA type of connections are shown in

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**Fig. 14.** Time history displacement with various connection types according to the (a) first order analysis and (b) second order analysis.
5.2. Single-bay two-storey frame

For the purpose of comparison of the analysis in this paper with existing computational methods the single-bay two-storey frame shown in Fig. 19 has been analyzed. Vibration and transient response analysis of the frame were investigated by Chan and Ho [25] and Chan and Chui [36]. They applied the numerical model based on the linearized second order theory assuming the linear and nonlinear types of connections. The stiffness and geometric matrices of the uniform beam with end springs were obtained using conventional cubic Hermitean shape functions. Two elements per beam and one element per column were applied. The flush end plate flexible connection type was assumed and modelled by Chen–Lui exponential model [39] shown in Fig. 19b.
The transient response of the frame was performed for the two cases of lateral loads (cyclic and impact) with and without the presence of gravitational loads. The same frame has been analyzed by the present numerical model and the results compared with those previously obtained by Chan and coworkers [25, 36].

Fig. 17. Time history acceleration with various connection types according to the (a) first order analysis and (b) second order analysis.

Fig. 18. Hysteresis $M-\theta$ loops at joint C of the frame with TSDWA (a) and DWA (b) type of connections.
The natural frequencies of the frame for the fully rigid and linear semi-rigid (flush end plate and TSDWA) connections have been determined and shown in Table 3. It can be seen that they are agree well with the results by Chan and Ho.

The time histories of the displacement at node B and the hysteretic loops at node A of the frame under the lateral cyclic loads obtained by Chan and Chui and in the current study are shown in Fig. 20. It can be seen that the response curves are very close.

Displacement response of the frame subjected to the impact loads obtained by Chan and Chui and by the present study is shown in Fig. 21. It can be seen that the response curves have the same character, but there are some differences between their amplitudes and periods. These differences are small at the beginning and they gradually increase with time. As expected, they are larger in the case the presence than in the case the absence of gravitational loads. The frame analyzed by the present study has smaller amplitudes and nonrecoverable deflections, as well as shorter periods when compared with the same frame analyzed by Chan and Chui.

### 6. Conclusion

An efficient method to perform dynamic analyses of steel frame structures with flexible connections has been presented in this paper. A numerical model that includes both nonlinear connection behavior and geometric nonlinearity of the structure has been developed. The complex dynamic stiffness matrix for a prismatic beam with rotational springs and dashpots attached at its ends was obtained in an explicit form. The stiffness matrix was based on the analytical solutions of the second order equations, so that each beam corresponds to one finite element.

On the bases of the above theoretical considerations and the results of the applied numerical analysis, it is evident that the flexible joint connections greatly influence the dynamic behavior of steel frames. The connection flexibility may significantly alter both vibration and the response of frames. An increase in the connection flexibility reduces the frame stiffness, and thus the eigenfrequencies, particularly the lower ones, which may have a primary influence on dynamic response of the structure.

From the results of numerical examples it can be concluded that the structural responses of the frames with nonlinear connections and the frames with conventional type of connections (rigid or linear) are considerably different. It shows that the effect of hysteretic damping on structural response is significant. Therefore, the nonlinear constitutive model for connections should be used in design and response analysis of real frame structures. The linear model is inadequate as it cannot represent a hysteretic behavior of connection under cyclic loads.

From the results, it can also be concluded that the viscous damping at connections may considerably reduce the displacement response and internal forces of the frame, particularly in the case of weak connection types. The influence of the geometric nonlinearity increases with the gravitational loads and the lateral frame deflections. It is higher in the frame with flexible connections than with rigid joints.

The connections are vital structural components that are very often responsible for the behavior and safety of frame structures subjected to strong dynamic (seismic) loads. Therefore, connection design and modelling have a great practical importance.
Appendix A.

The interpolation functions $N_i(x)$, $i = 1, \ldots, 4$, for the compressive member ($N < 0$), are:

$N_1(x) = \Delta^{-1}[1 - \cos \omega - \omega \xi \sin \omega + \sin \omega \sin \omega \xi$
$- \sin \omega \sin \omega \xi + (1 - \cos \omega) \cos \omega \xi],$

$N_2(x) = I(\Delta \omega)^{-1}[\omega \cos \omega - \sin \omega + \omega \xi(1 - \cos \omega)$
$+ (1 - \cos \omega - \omega \sin \omega) \sin \omega \xi$
$+ (\sin \omega - \omega \cos \omega) \cos \omega \xi],$

$N_3(x) = \Delta^{-1}[1 - \cos \omega - \omega \xi \sin \omega + \sin \omega \sin \omega \xi$
$- (1 - \cos \omega) \cos \omega \xi],$

$N_4(x) = I(\Delta \omega)^{-1}[\sin \omega - \omega + \omega \xi(1 - \cos \omega)$
$- (1 - \cos \omega) \sin \omega \xi + (\omega - \sin \omega) \cos \omega \xi],$

where

$\Delta = 2(1 - \cos \omega) - \omega \sin \omega, \quad \xi = \frac{x}{l}, \quad \omega = \sqrt{\frac{N}{EI}}.$
The shape functions for the tensile member \((N > 0)\) can be obtained from the foregoing expressions replacing \(\omega = j\omega\) and using the relations \(she = -j\sin j\omega\) and \(che = \cos j\omega\).

The elements of the correction matrix \(G\) are:

\[
\begin{align*}
g_{21} &= -g_{23} = -\frac{6}{I}\left[ g_1 + 2g_1g_2(2\phi_3 - \phi_4)\right] \phi_2, \\
g_{22} &= \frac{-6e_1}{IA} \left[ g_1 + 2g_1g_2(2\phi_3 - \phi_4)\right] \phi_2 \\
&- 4\left[ g_1\phi_3 + g_1g_2(4\phi_3^2 - \phi_4^2)\right], \\
g_{24} &= -\frac{6e_2}{IA} \left[ g_1 + 2g_1g_2(2\phi_3 - \phi_4)\right] \phi_2 - 2g_1\phi_4, \\
g_{41} &= -g_{43} = -\frac{6}{I}\left[ g_2 + 2g_1g_2(2\phi_3 - \phi_4)\right] \phi_2, \\
g_{42} &= \frac{-6e_1}{IA} \left[ g_2 + 2g_1g_2(2\phi_3 - \phi_4)\right] \phi_2 - 2g_2\phi_4, \\
g_{44} &= \frac{-6e_2}{IA} \left[ g_2 + 2g_1g_2(2\phi_3 - \phi_4)\right] \phi_2 \\
&- 4\left[ g_2\phi_3 + g_1g_2(4\phi_3^2 - \phi_4^2)\right], \\
\Delta &= (1 + 4g_1\phi_3)(1 + 4g_2\phi_3) - 4g_1g_2\phi_4^2, \\
g_i &= \frac{EI}{lk_i}, \\
i &= 1, 2.
\end{align*}
\]

Fig. 21. Displacement response of two-storey frame under impact loads. (a) Without gravity and (b) with gravity loads.
The elements of matrix $k_1$ according to the second order theory are:

\[
k_{11} = -k_{113} = k_{133} = -\frac{EI}{l^2 A} 12\ell \phi_2(a + b),
\]
\[
k_{12} = -\frac{EI}{l^2 A} 4l^2 (2\phi_3 c + \phi_4 e),
\]
\[
k_{14} = -k_{134} = -\frac{EI}{l^2 A} [6l \phi_2(c + f) + 2l^2 (2\phi_3 a + \phi_4 b)],
\]
\[
k_{14} = -\frac{EI}{l^2 A} 2l^2 (2\phi_3 (e + f) + \phi_{45}(d + c)],
\]

where $k_{13} = k_{43}$.

The elements of matrix $k_2$ according to the second order theory are:

\[
k_{211} = -k_{213} = k_{233} = \frac{EI}{l^2 A} \left[ 4\phi_3(a^2 + b^2) + 4\phi_4 ab + \frac{a^2}{g_1} + \frac{b^2}{g_2} \right],
\]
\[
k_{212} = -k_{223} = \frac{EI}{l^2 A} \left[ 4\phi_3(ac + bf) + 2\phi_4(ab + ce) + \frac{ac}{g_1} + \frac{bf}{g_2} \right],
\]
\[
k_{214} = -k_{234} = \frac{EI}{l^2 A} \left[ 4\phi_3(ad + bd) + 2\phi_4(ad + ce) + \frac{ac}{g_1} + \frac{bd}{g_2} \right],
\]
\[
k_{22} = \frac{EI}{l^2 A} \left[ 4\phi_3(c^2 + f^2) + 4\phi_4 cf + \frac{c^2}{g_1} + \frac{e^2}{g_2} \right],
\]
\[
k_{24} = \frac{EI}{l^2 A} \left[ 4\phi_3(d^2 + e^2) + 4\phi_4 ed + \frac{c^2}{g_1} + \frac{d^2}{g_2} \right],
\]
\[
k_{24} = \frac{EI}{l^2 A} \left[ 4\phi_3(cf + df) + 2\phi_4(cf + df) + \frac{ce}{g_1} + \frac{df}{g_2} \right],
\]

where $a = g_{21} = -g_{23}$, $b = g_{41} = -g_{43}$, $c = g_{22}$, $d = g_{44}$, $e = g_{24}$, $f = g_{42}$ and $c_1 = c_2 = 0$.

Analytical expressions for the functions $\phi_i$, $i = 1, \ldots, 4$ can be found in Ref. [35].

The elements of complex dynamic stiffness matrix $k^*$ according to the first order analysis are:

\[
k_{11}^* = -k_{13}^* = k_{13} = \frac{12EI}{l^2 A} (1 + 2g_1^*),
\]
\[
k_{14}^* = -k_{34}^* = \frac{6EI}{l^2 A} (1 + 2g_1^*),
\]
\[
k_{12}^* = -k_{23}^* = \frac{6EI}{l^2 A} (1 + 2g_2^*),
\]
\[
k_{14}^* = \frac{4EI}{l^2 A} (1 + 3g_1^*),
\]
\[
k_{12}^* = \frac{4EI}{l^2 A} (1 + 3g_2^*),
\]
\[
k_{14}^* = \frac{4EI}{l^2 A},
\]
\[
k_{k}^* = \frac{1}{l} k_l^*,
\]

where $j = \sqrt{-1}$, $\Delta^* = 1 + 4g_1^* + 4g_2^* + 12g_1^* g_2^*$. The elements of damping matrix $c$ are:

\[
c_{11} = -c_{13} = c_{33} = \frac{36EI}{l^2 A^2} (h_1 + h_2 + 4g_1 h_2 + 4g_1^* h_1 + 4g_2 h_1 + 4g_2^* h_1),
\]
\[
c_{12} = -c_{23} = \frac{12EI}{l^2 A^2} (2h_1 + h_2 + 2g_1 h_2 + 10g_1 h_1 + 12g_2^* h_1),
\]
\[
c_{14} = -c_{34} = \frac{12EI}{l^2 A^2} (h_1 + 2h_2 + 10g_1 h_2 + 12g_2 h_2 + 4g_2 h_1),
\]
\[
c_{22} = \frac{4EI}{l^2 A} (4h_1 + h_2 + 24g_2 h_1 + 36g_1^* h_1),
\]
\[
c_{24} = \frac{8EI}{l^2 A} (h_1 + h_2 + 3g_1 h_2 + 3g_2 h_1),
\]
\[
c_{44} = \frac{4EI}{l^2 A} (h_1 + 4h_2 + 24g_2 h_1 + 36g_1^* h_1),
\]

where $h_i = (c_i EI)/(l k^*_l)$, $i = 1, 2$.

The elements of mass matrix $m$ are:

\[
m_{11} = -m_{13} = m_{33} = \frac{36EI}{l^{3/2} A g_1 g_2} (g_2 h_1^2 + 8g_2^2 h_1^2 + 20g_1^2 h_1^2 + 16g_1^4 h_1^2)
\]
\[
- 4g_1 g_2 h_1 h_2 - 8g_1^2 g_2 h_1 h_2 - 8g_1 g_2^2 h_1 h_2
\]
\[
- 16g_1^3 g_2 h_1 h_2 + g_1^2 h_2^2 + 8g_1^2 h_2^2 + 20g_1^3 h_2^2 + 16g_1^4 h_2^2),
\]
\[
m_{12} = -m_{33} = \frac{12EI}{l^{3/2} A g_1 g_2} (2g_2 h_1^2 + 18g_2^2 h_1^2 + 52g_1^2 h_1^2 + 48g_1^4 h_1^2)
\]
\[
- 6g_1 g_2 h_1 h_2 - 8g_1^2 g_2 h_1 h_2 - 16g_1 g_2^2 h_1 h_2
\]
\[
- 24g_1^3 g_2 h_1 h_2 + g_1^2 h_2^2 + 6g_1^3 h_2^2 + 8g_1^4 h_2^2),
\]
\[
m_{14} = -m_{34} = \frac{12EI}{l^{3/2} A g_1 g_2} (g_2 h_1^2 + 6g_2^2 h_1^2 + 8g_1^2 h_1^2)
\]
\[
- 6g_1 g_2 h_1 h_2 - 16g_1^2 g_2 h_1 h_2 - 8g_1^2 g_2^2 h_1 h_2
\]
\[
- 24g_1^3 g_2 h_1 h_2 + 2g_1^2 h_2^2 + 18g_1^3 h_2^2 + 52g_1^4 h_2^2
\]
\[
+ 48g_1^5 h_2^2),
\]
\[ m_{22} = \frac{4EI}{LA^3 g_1 g_2} \left( 4g_1 h_1^2 + 40g_2 h_2^1 + 132g_1 h_1^2 + 144g_2 h_2^2 ight) - 8g_1 g_2 h_1 h_2 - 24g_1 g_2 h_1 h_2 + g_1 h_2^2 + 4g_2 h_2^2 \]

\[ m_{24} = \frac{8EI}{LA^3 g_1 g_2} \left( g_1 h_1^2 + 7g_2 h_2^2 + 12g_1 h_1^2 - 5g_1 g_2 h_2 ight) - 12g_1 g_2 h_1 h_2 - 12g_1 g_2 h_1 h_2 - 36g_1 g_2 h_1 h_2 + g_1 h_2^2 + 7g_2 h_2^2 + 12g_1 h_2^2 \]

\[ m_{44} = \frac{4EI}{LA^3 g_1 g_2} \left( g_1 h_1^2 + 4g_2 h_2^2 - 8g_1 g_2 h_1 h_2 - 24g_1 g_2 h_1 h_2 + 4g_1 h_2^2 + 40g_2 h_2^2 + 132g_1 h_2^2 + 144g_2 h_2^2 \right) \]

\[ m_{jk} = m_{kj}. \]

References


