Multiuser subcarriers and bit allocation combining adaptive mapping for SC-FDMA system

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Abstract

In this article, a multiuser single carrier frequency division multiple access (SC-FDMA) system is considered, based on which an adaptive subcarrier and bit allocation algorithm is investigated. The algorithm has been used to achieve a subcarrier mapping mode in this system, which combines the advantages of single- and multi-carrier transmissions, such as, low peak to average power ratio, orthogonality of signals of different users, and low complexity. Simulation results show that it has a similar performance as that of the adaptive allocation algorithm in the orthogonal frequency division multiple (OFDM) system and the proposed mapping mode has a performance gain over the two existing mapping modes at the link level.

Keywords adaptive subcarrier and bit allocation, SC-FDMA, subcarrier mapping modes, bit error ratio (BER)

1 Introduction

At present, research on beyond third generation (B3G) mobile radio system is in progress worldwide. A future mobile radio system has to meet challenging requirements. A well-established representative is orthogonal frequency division multiple access (OFDMA). It provides low computational complexity and at the same time good performance [1]. However, OFDMA signals are sensitive to frequency offsets and suffer from high envelope fluctuations. Another solution is the SC-FDMA system, which utilizes single carrier modulation. It is a technique that has a similar performance and essentially the same overall complexity as that of the OFDM system [2]. An outstanding advantage of SC-FDMA is its lower peak to average power ratio (PAPR), because of its inherent single carrier structure. SC-FDMA has drawn great attention as an attractive alternative to OFDMA, especially in the uplink communications where lower PAPR greatly benefits the mobile terminal, in terms of transmit power efficiency [3].

In the SC-FDMA systems, there are two methods to choose subcarriers for transmission: distributed subcarrier mapping mode and localized subcarrier mapping mode. The comparison of the link level performance between OFDMA and SC-FDMA, using the two mapping modes has been proposed in Ref. [2]. However both of them do not consider the channel state to allocate the carrier. In this article, a subcarrier mapping mode based on an adaptive subcarrier and bit allocation algorithm is investigated. If the transmitter is aware of the channel state information (CSI), the transmitted signal can avoid the spectral nulls and deep depressions of the channel. Thus, the authors will use CSI and an adaptive subcarrier and bit allocation algorithms to choose the subcarrier mapping mode, to decrease the symbol error probability.

The remainder of this article is organized as follows. Section 2 gives an overview of SC-FDMA systems. Section 3 derives the mapping selected method based on adaptive subcarrier and bit allocation algorithm. The simulation results of BER performance are presented in Sect. 4. Finally, some conclusions are drawn in Sect. 5.

2 Overview of SC-FDMA system

A block diagram of an SC-FDMA system is shown in Fig. 1. The portion of subcarrier and bit allocation is proposed in the next section, which is the proposed adaptive subcarrier mapping mode. SC-FDMA can be regarded as a discrete Fourier transform (DFT)-spread OFDMA, where time domain data symbols are transformed to frequency domain by DFT before going through the OFDMA modulation. The orthogonality of all users, stems from the fact that each user
occupies different subcarriers in frequency domains, similar to the case of OFDMA, which is shown in Fig. 2.

To analyze the performance of SC-FDMA, the transmitter signal of one user $u$ without cyclic prefix (CP) will be written as:

$$S_u = F_u^H T_{N,M} F_M D_u$$  

(1)

where $D_u = [d_{u,0}, d_{u,1}, \ldots, d_{u,M-1}]^T$ represents the transpose operation and $d_{u,i}$ is the modulated symbol. $F_M$ is $M$-point fast Fourier transform (FFT) and $F_N^*$ is the $N$-point inverse fast Fourier transform (IFFT) matrix. $[^T]$ represents the transpose operation. Where

$$F_M = [f_1^T \ f_2^T \ \ldots \ f_M^T]^T$$

$$f_i = \frac{1}{\sqrt{M}} [0 \ \ e^{-j2\pi i/M} \ \ldots \ e^{-j2\pi (M-1)i/M}]$$

$T_{N,M}$ is the mapping matrix of user $u$ for subcarrier assignments and its values are decided by subcarrier allocation.

Through the fading channel, reverse processing will be done at the receiver’s side. After removing the guard interval and going through the $N$-point FFT, the received signal of user $u$ at the frequency domain is expressed as:

$$R_u = H_u T_{N,M} F_M D_u + n$$  

(2)

where $H_u = \text{diag}(h_{uk})$ and $h_{uk}$ is the frequency channel response at subcarrier $k$ of user $u$. $n$ is the additive white Gaussian noise (AWGN) vector and $R_u = [r_u(0), r_u(1), \ldots, r_u(N-1)]^T$, in which $r_u(k)$ is the received signal at subcarrier $k$.

There are two methods to decide $T_{N,M}$ as shown in Fig. 3. In the distributed subcarrier mapping mode, DFT outputs of the input data are allocated over the entire bandwidth with zeros occupying the unused subcarriers, whereas, consecutive subcarriers are occupied by the DFT outputs of the input data in the localized subcarrier mapping mode.

In the next section, the authors propose an adaptive subcarrier and bit allocation method to select the mapping mode according to CSI.

3 Selection mapping mode based on adaptive subcarrier and bit allocation

For the distributed mode, the frequency samples $\{X_i\}$
The authors assume that the subcarriers used for SC-FDMA to choose the mapping mode to combine the fading channel. Consequently, after subcarrier mapping, researchers have developed many bit and power allocation algorithms [4–8]. Subsequently, the authors can use CSI and the adaptive allocation algorithm to find the optimal assignment of \( P_u, u \) according to:

\[
\begin{align*}
P_u &= P_u + |g_{u,k}|^2 \\
I &= I - I_k \\
C_{u,s} &= k \\
s &= s + 1
\end{align*}
\]

d) Go to the next user in the short list determined in step a) until all users are allocated another subcarrier.

### 3.2 Bit loading

Now to consider bit loading under the assumption that subcarrier allocation is completed. After subcarrier allocation, the authors use bit loading on every selected subcarrier. The authors denote \( f(c_u(k)) \) as the required received power per symbol in the \( k \)th subcarrier of user \( u \) to satisfy a given BER requirement in a \( c \) bit/symbol modulation scheme. Assuming that the channel state information is known, the transmission power of the \( k \)th subcarrier can be given by:

\[
p_u(k) = \frac{f(c_u(k))}{|h_{u,k}|^2}
\]

where \( h_{u,k} \) is the channel gain of user \( u \) and the \( k \)th subcarrier. Thus, the combined bit and power allocation should find the optimal assignment of \( c_u(k) \), so that the total transmission power is minimized, satisfying the transmission rate and BER \( V_{BER} \) requirements. The optimization problem can be mathematically formulated as follows:

\[
\begin{align*}
\min_{c_u(k)=0, 1, \ldots, U} & \quad P_T \\
\text{s. t.} & \quad \sum_{k=1}^{U} c_u(k)\rho_{u,k} = B_s \\
& \quad u = 1, 2, \ldots, U
\end{align*}
\]

(7)

where \( D \) is the set of all possible values for \( c_u(k) \), and \( c_u(k) = 0 \) means that no information is transmitted through the \( k \)th subcarrier. As in Ref. [4], the function \( f(c_u(k)) \) is defined as

\[
f(c_u(k)) = \frac{N_0}{3} \left[ Q^{-1}\left( V_{BER}^{\frac{3}{4}} \right) \right]^2 (2^{c_u(k)} - 1)
\]

(8)

where the authors recall that

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt
\]

The authors assume that the subcarriers used for SC-FDMA to choose the mapping mode to combine the fading channel. Consequently, after subcarrier mapping, a two-step approach to solve the allocation problem. In the first step, subcarriers assigned to each user can achieve the adaptive mapping mode. Next, bits are allocated on every subcarrier.

### 3.1 Subcarrier allocation

An adaptive dynamic subcarrier allocation (DSA) algorithm is proposed in Ref. [4]. The authors have revised this algorithm and realized the subcarrier allocation.

In the following, \( P_u \) represents the average received power for user \( u \), \( U \) is the total number of users, \( I \) is a set containing the indices of the usable subcarrier. \( M \) is DFT size, that is, the number of allocated subcarriers for each user. \( h_{u,k} \) is the channel gain for user \( u \) and subcarrier \( k \). \( C_{u,s} \) is an element in matrix \( C \) to record the location of the allocated subcarriers for user \( u \) and subcarrier \( s \).

**Initialization**

- Set \( P_u = 0 \) for all users \( u = 1, 2, \ldots, U \)
- Set \( C_{u,s} = 0 \) for all users \( u = 1, 2, \ldots, U \)
- Set \( s = 1 \)

**Main process**

- For \( l = 1 : M \)
  - a) Make a short list according to the users that have less power. Find user \( u \) satisfying \( P_u \leq P_i, 1 \leq i \leq U \).
  - b) For the user \( u \) chosen in a), find subcarrier \( k \) satisfying \( |h_{u,k}| \geq |h_{u,i}|, 1 \leq j \leq N \).
  - c) Update \( P_u, I \) and \( C_{u,s} \) with the \( k \) from step b) according to:
    \[
    P_u = P_u + |g_{u,k}|^2 \\
    I = I - I_k \\
    C_{u,s} = k \\
    s = s + 1
    \]
  - d) Go to the next user in the short list determined in step a) until all users are allocated another subcarrier.

After subcarrier mapping \( T_{k,u} \) can be described as follows:

\[
\hat{X}_k = \begin{cases} X_k; & k = QI \ (0 \leq I \leq M - 1) \\ 0; & \text{otherwise} \end{cases}
\]

where \( Q = N / M \).

And for the localized mode, the frequency samples after subcarrier mapping \( \{\hat{X}_k\} \) can be described as follows:

\[
\hat{X}_k = \begin{cases} X_k; & 0 \leq k \leq M - 1 \\ 0; & M < k \leq N - 1 \end{cases}
\]

where \( \{X_k : l = 0, 1, \ldots, M - 1\} \) are the frequency domain samples after DFT and \( \{\hat{X}_k : k = 0, 1, \ldots, N - 1\} \) are frequency domain samples after subcarrier mapping.

As different subcarriers experience different fades and transmit different numbers of bits, the transmit power levels must be changed accordingly. Researchers have developed many bit and power allocation algorithms [4–8]. Subsequently, the authors can use CSI and the adaptive allocation algorithm to achieve the mapping mode to combine the fading channel. The authors assume that the subcarriers used for SC-FDMA transmission are from the set \( \kappa = \{k_1, k_2, \ldots, k_N\} \subset [1, N] \), therefore, after subcarrier mapping, \( T_{k,u} \), \( \{\hat{X}_k\} \) can be described as follows.

\[
\hat{X}_k = \begin{cases} X_k; & k \in \kappa \\ 0; & \text{otherwise} \end{cases}
\]

To achieve the mapping mode, the authors consider a two-step approach to solve the allocation problem. In the first step, subcarriers assigned to each user can achieve the adaptive mapping mode. Next, bits are allocated on every subcarrier.

- a) Make a short list according to the users that have less power. Find user \( u \) satisfying \( P_u \leq P_i, 1 \leq i \leq U \).
- b) For the user \( u \) chosen in a), find subcarrier \( k \) satisfying \( |h_{u,k}| \geq |h_{u,i}|, 1 \leq j \leq N \).
- c) Update \( P_u, I \) and \( C_{u,s} \) with the \( k \) from step b) according to:
  \[
  P_u = P_u + |g_{u,k}|^2 \\
  I = I - I_k \\
  C_{u,s} = k \\
  s = s + 1
  \]
- d) Go to the next user in the short list determined in step a) until all users are allocated another subcarrier.
and $\rho_{k, i}$ is an indicator variable defined as

$$
\rho_{k, i} = \begin{cases} 
0, & \text{if } c_i(k) = 0 \\
1, & \text{otherwise}
\end{cases}
$$

As the power needed to transmit a certain number of bits in a subcarrier is independent of the numbers of bits allocated to other subcarriers, it turns out that a greedy approach is optimal. A greedy algorithm assigns bits to the subcarriers, one bit at a time, and in each assignment, the subcarrier that requires the least additional power is selected. The bit allocation process will be completed when all $R$ bits are assigned. Several articles have provided various algorithms for the problem. The basic structure of most algorithms are similar and can be described as follows [5]:

Initialization

$c_i(0) = 0, \forall k, u$

$\Delta p_{k, u}(c_i) = \frac{f(1) - f(0)}{g_{k, i}}, \forall k, u$

For each $u$, repeat the following $B_u$ times:

$\hat{k} = \arg \min_{k \in \mathcal{K}} \Delta p_{k, u}(c_i(k))$

$c_i(\hat{k}) = c_i(\hat{k}) + 1$

$\Delta p_{k, u}(c_i(\hat{k})) = \frac{f(c_i(\hat{k}) + 1) - f(c_i(\hat{k}))}{g_{k, i}}$

Finish

Bit allocation : $c_i(k), \forall k, u$

Power allocation : $p_{k, u}(k) = \frac{f(c_i(k))}{g_{k, i}}, \forall k, u$

The initialization stage computes, for each subcarrier, the additional power needed to transmit an additional bit. For each iteration bit assignment, the subcarrier that needs the minimum additional power is assigned one more bit, and the new additional power for that subcarrier is updated. After $B_u$ iterations, the final bit assignment gives the optimal bit allocation for each subcarrier. It is important to note that the bit allocation is optimal only for the given function $f(c_i(k))$, which depends on the selected modulation scheme.

After completion of the above-mentioned procedure, the authors can get the mapping mode $T_{N, M}$ according to the subcarrier allocation algorithm, based on CSI, and the adaptive modulation according to the bit loading.

4 Simulation results

In this section, the link level simulation results are compared. To evaluate the proposed adaptive mapping mode based on subcarrier and bit allocation algorithm of SC-FDMA systems, the authors assume that the channel has not changed within one OFDM symbol interval. Then the equalization also can be done in the frequency domain for SC-FDMA. In this article, the authors adopted a minimal mean square error (MMSE) equalizer. After MMSE operation, the signal will be:

$$
\hat{\mathbf{r}}_u = H_u W \mathbf{H}_T \mathbf{T}_{N, M} F_u \mathbf{D}_u + \mathbf{n}
$$

Here the matrix $W$ is

$$
W = \text{diag} \left( \frac{1}{\sigma^2 (k) + \frac{1}{SNR}} \right)
$$

where signal to noise ratio (SNR) refers to the signal to noise ratio of subcarrier $k$. Then the signal after $M$-point IFFT is written as:

$$
\hat{\mathbf{D}}_u = F_u \mathbf{T}_{N, M} \hat{\mathbf{r}}_u
$$

Now the data $\hat{\mathbf{D}}_u$ can be forwarded for soft demodulation.

Table 1 shows the system parameters used in the simulation. To simplify the simulation process, the authors assume that there are four users in the system, that is, all users can transmit data in each allocation process. White noise was assumed, that is $\sigma^2 = N_0, \forall n$. To evaluate the performance of this scheme, the authors have simulated a three-path frequency selective Rayleigh fading channel with an exponential power delay profile and Pedestrian A with parameters of 3 km/h speed and a four-path Jakes fading model.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Simulation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Values</td>
</tr>
<tr>
<td>Carrier frequency/GHz</td>
<td>2.0</td>
</tr>
<tr>
<td>Channel bandwidth/MHz</td>
<td>1.25</td>
</tr>
<tr>
<td>Subcarrier number ($N$)</td>
<td>64</td>
</tr>
<tr>
<td>Cycle prefix</td>
<td>16</td>
</tr>
<tr>
<td>Pre-DFT size</td>
<td>16</td>
</tr>
<tr>
<td>Frame length $T$ms</td>
<td>2</td>
</tr>
<tr>
<td>Symbols per frame</td>
<td>5</td>
</tr>
<tr>
<td>$M$QAM available</td>
<td>1, 2, 4</td>
</tr>
</tbody>
</table>

To demonstrate the adaptive allocation, an instance of the channel was generated, based on the exponential power delay profile, and the subcarrier mapping mode was found. Figure 4 shows the channel frequency response of this channel model and the allocation of bits to each subcarrier for all users, based on which the authors have achieved the optimal adaptive mapping mode.

As expected, as to one user when the channel has high frequency response, the data after DFT mode was mapped on it to transmit and the subcarriers experiencing very poor channel instances had zero bits allocated to them.
Simulation results for the average raw (uncoded) BER performance of the SC-FDMA system, considered both with and without the adaptive mapping mode, are presented in Fig. 5. At the same time, the link level simulation results of the two multiple access schemes of OFDMA and SC-FDMA for uplink are compared. As for the OFDMA scheme, adaptive multi-user dynamic subcarrier allocation algorithm has been proposed previously in Refs. [4, 6]. In this article, the authors compared the performance of the two schemes.

5 Conclusions

This article has presented a subcarrier mapping mode by using an adaptive subcarrier and bit allocation algorithm in multiuser SC-FDMA systems. This method combines the advantages of single- and multi-carrier transmissions, such as, low peak to average power ratio, orthogonality of signals of different users, and low complexity. Simulation results show that the BER performance has been approved, compared with the two existing mapping modes.

Acknowledgements

This work is supported by the ZTE Research Foundation.

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scheme is based on SCH and CSRSs, which are for synchronization and cell identification respectively. Moreover, CSRSs are also used for fine CFO estimation. The cell search algorithm is given in detail. The proposed method can satisfy the requirements of LTE search and the simulations show that it is effective.

Acknowledgements

This work is supported by the cooperative research between Beijing University of Posts and Telecommunications and TD Technology Ltd.

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