Chapter 5

PHASE-SHIFTING DIGITAL HOLOGRAPHY

Principles and applications

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Abstract: In digital holography holograms are recorded by a CCD and image reconstruction is performed by a computer. It is free from tedious photographic processing and delivers three-dimensional distributions of both amplitude and phase quantitatively. Its main limitation that is caused by much lower resolution of CCDs than photographic materials has been substantially overcome by phase-shifting digital holography that reduces the spatial frequency of hologram by employing the in-line setup and directly evaluates complex amplitude at the CCD plane to eliminate the conjugate and zero-th order images appearing in the off-axis setup which has been commonly used. In this chapter we first describe its basic principle of image formation. It is followed by applications to microscopy, color holography, and data compression for storage, transmission, and real-time display of holographic data. Then its applications to measurements of shape and deformation of diffusely reflecting surfaces are discussed in comparison with conventional holographic interferometry and electronic speckle pattern interferometry.

Key words: Interferometry, holography, image processing, microscopy, color image, data compression, speckle, shape and deformation measurement

1. INTRODUCTION

Recent remarkable and quick advances in digital cameras and computers have been substantially improving practical utilities of digital holography that uses digital recording of holograms and digital reconstruction of 3-dimensional images. Especially, drastic increases of pixel numbers of CCD
and computational power have enabled us to produce the reconstructed images with higher quality in only a few seconds. For guaranteeing the quality of the reconstructed image it is necessary to use as many pixels for the final image as possible. In the off-axis setup that was first used in digital holography the reconstructed image is accompanied by the zero-order and the conjugate images and hence only one-ninth of the total pixels are utilized for the final image. It is due to direct use of the interference intensity leading to ambiguity of the related phase distribution. This ambiguity can be eliminated by directly deriving the complex amplitude of the object wave by means of phase-shifting interferometry, which has been mainly applied to analysis of phase objects and optical surfaces. However, it can also be used for derivation of complex amplitude when both the real amplitude and phase show variations.

The phase-shifting digital holography was first successfully applied to diffusely reflecting objects for which both the amplitude and phase distributions exhibit random speckle-like variations. The image reconstruction process also becomes simpler and straightforward because no spatial frequency filtering is necessary as in the off-axis method. In this paper we first discuss image formation by phase-shifting digital holography that is then applied to 3-d microscopy, color holography, and data compression. Then we describe measurement of surface shape and deformation of diffusely reflecting surfaces using phase-shifting digital holography in conjunction with the conventional methods such as holographic and speckle interferometry. The latter field can be regarded as an extension of conventional phase-shifting techniques that are applied for holographic and speckle interferometry. The present method requires simpler setups with an aid of a computer not only in phase analysis but also in imaging where phase information is essential. Thus phase-shifting digital holography realizes a new flexibility of quantitative acquisition, processing, transmission, and display of coherent optical information by means of modern digital processing and communication channels and optoelectronic devices.

2. IMAGE FORMATION

2.1 Hologram recording and analysis

The basic setup for phase-shifting digital holography is illustrated in Fig.5-1. A laser beam is divided into two paths. One beam is expanded to illuminate an object. The light scattered from the object is combined at the
Phase-Shifting Digital Holography

CCD with the collimated reference beam after reflection at the PZT mirror controlled by a computer through a beam splitter to ensure the in-line configuration. At least three interference patterns are acquired after stepwise phase shifts of the reference beam. For analysis of image formation we adopt the coordinate system depicted in Fig.5-2. The object wave at the CCD plane is represented as a Fresnel transform of the complex amplitude at the object plane \( U_o(x', y') \) by

\[
U(x, y) = \int \int U_o(x', y') \exp \left[ ikz_o + ik \frac{(x-x')^2 + (y-y')^2}{2z_o} \right] dx' dy'
\]

(5-1)

where integration is carried out over infinity and \( z_o (>)0 \) is the distance from the object plane to the CCD. The interference intensity detected by the CCD is given by

\[
I_H(x,y; \delta) = |U_R(x,y) \exp(i\delta) + U(x,y)|^2 = |U_R|^2 + |U|^2 + 2 \Re [U_R U^* \exp(i\delta)]
\]

(5-2)

where \( U_R(x,y) \) is the complex amplitude of the reference beam and \( \delta \) is the mean phase difference between the object and the reference waves. By using the phase-shifting procedure in which the reference phase is shifted by a step of \( \pi/2 \) at least three times, we can derive the complex amplitude of the object wave such as

\[
U(x,y) = \frac{1}{4U_R} \left\{ I_H(x,y; 0) - I_H(x,y; \pi) + i[I_H(x,y; \pi/2) - I_H(x,y; 3\pi/2)] \right\}
\]

(5-3)

in the case of four-step algorithm, and

\[
U(x,y) = \frac{1-i}{4U_R} \left\{ I_H(x,y; 0) - I_H(x,y; \pi/2) + i[I_H(x,y; \pi/2) - I_H(x,y; \pi)] \right\}
\]

(5-4)

in the three-step algorithm. If we compare these expressions with conventional phase analysis using the phase-stepping method, we find it rather unique that we can treat the whole process with complex numbers also in numerical computations.
2.2 Image reconstruction

The reconstruction is performed by the Fresnel transformation of the derived complex amplitude in such a way that

\[ U_I(X, Y, Z) = \iiint U(x, y) \exp \left[ i k z + ik \frac{(X-x)^2 + (Y-y)^2}{2Z} \right] dx dy \]  

where the integration is carried out over the area of CCD. We assume first the sufficient extension and ideal resolution of the device. Here the collimated reference beam is assumed with \( z_R = \infty \). If we substitute Eq. (5-1) into Eq. (5-5), we find that the image plane is determined from the condition that the quadratic term of \( x \) and \( y \) in the exponent vanishes such as \( Z = -z_o \) where the complex amplitude becomes

\[ U_I(X, Y, Z) = U_o(x', y') \]
\[ U_1(x, y, -z_0) = U_0(x, y) \tag{5-6} \]

if we neglect limitation of the finite size of CCD array. Image reconstruction using Eq. (5-5) can be numerically accomplished by regarding it as either a Fourier transform or a convolution integral and replacing the integration by summation. In the former algorithm to be called mow the single FFT method, the sample interval of the image is given by \( \lambda Z/L \) where \( L \) is the size of CCD, while the latter, named here the double FFT method because it uses FFT twice, keeps the sampling interval equal to be the pixel pitch of CCD independent of the reconstruction distance. Recently relationships between these algorithms have been clarified together with proposal of a new algorithm using double Fresnel transforms\(^6\). It is especially useful in color digital holography where reconstructed image at three wavelengths are superposed\(^7, 8\). In the case of diffusely reflecting objects only the phase distribution \( \arg\{U(x, y)\} \) is enough to reconstruct images of almost the same quality as will be mentioned below in more detail\(^9, 10\).

If we record an object by using a geometry shown in Fig.5-1, the resolution is given by

\[ \Delta x = \lambda |z_0| / L \tag{5-7} \]

while the focal depth of the reconstructed image is represented by

\[ \Delta z = 2\lambda z_0^2 / L^2 \tag{5-8} \]

The maximum object size to be recorded is equal to \( N\Delta x \) with the pixel number \( N \) along the x-direction in the single FFT algorithm, while it becomes equal to that of CCD in the double FFT method. The effects of CCD parameters on image resolution were investigated numerically\(^11\).

### 2.3 Three-dimensional microscopy

For attaining higher resolution we can insert a microscope objective between the object and the CCD and the defocused image of the object is superposed with the reference wave at the CCD. The reconstruction is carried out by the Fresnel transformation. In this case we can attain the same magnification and resolution as obtained from direct imaging with the objective\(^12\). The second setup for microscopy employs a divergent reference as shown in Fig.5-3. The confocal system where the reference point source and the object are located at nearly the same distance from the CCD is employed to reduce the spatial frequency of the hologram\(^11\). In this case we
have to multiply the integrand of Eq.(5-5) by the parabolic phase function expressed by

\[ U_M(x,y) = \exp \left( ik \frac{x^2 + y^2}{2z_M} \right) \tag{5-9} \]

with the distance \( z_M \) of the point reference from the CCD plane. In conventional imaging by a lens, this parabolic phase function is provided by a lens. The coefficient of the quadratic term of \( x \) and \( y \) in the exponent of Eq.(5-5) vanishes at

\[ Z = Z_I = -1 \sqrt{\left( \frac{1}{z_Q} + \frac{1}{z_R} \right)} \tag{5-10} \]

The condition that the linear term of Eq.(5-5) vanishes leads to the magnification factor given by

\[ m = \left( 1 + \frac{z_Q}{z_M} \right)^{-1} \tag{5-11} \]

Although this property is well known in conventional holography, it can be used more flexibly in digital holography. Figure 5-4 shows examples of the amplitude and phase images of onion peels both focused and defocused\(^\text{11}\). The wavelength is 514 nm of an Ar-laser. Instead of a CCD camera we employed a CCD mounted on a card (SONY CCB-M27B, 768x494 pixels, 6.45x4.84 \( \mu \)m\(^2\)) so that the cubic beam splitter (5x5x5 mm\(^3\)) could approach the device as close as possible. Four phase-shifted images were first stored in a frame memory with 512x512x8 bits. The focused intensity image exhibits the same quality as that taken with a microscope. In the phase images the optical thickness distribution of the transparent parts of cells can be found. It has a saw-tooth cross-section to deliver the distribution after phase-unwrapping. The resolution observed from a test target proved to be 3 \( \mu \)m, while the theoretical value was 2.5 \( \mu \)m. Thus by phase-shifting digital holography we can realize a 3-dimensional microscopy applicable for various kinds of objects with a simple setup and without any mechanical movement except for phase-shifting as well as straightforward image reconstruction without frequency filtering needed for off-axis configuration to separate the desired image.

For digital holographic microscopy using complex amplitude information we also can mention scanning holographic microscopy that detects the complex amplitude by optical heterodyne technique where the reference
beam is frequency shifted and an object is scanned by an X-Y stage under focused illumination\textsuperscript{13}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5-3.png}
\caption{Setup for microscopy using phase-shifting digital holography without imaging lens}
\end{figure}

**Onion peels**

\[ Z_R = 17\text{mm}, \ Z_0 = 17\text{mm}, \ \text{Magnification} \ m = 5.6, \ \lambda = 514.5\text{nm} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5-4.png}
\caption{Reconstructed images of onion peels at various reconstruction distances}
\end{figure}
Phase-shifting digital holography was also applied to color holography by using a multi-line laser and a color CCD shown in Fig.5-5. With the phase-shifting of the reference beam in-line holograms for three wavelengths emitted from a He-Cd laser are recorded simultaneously to derive the complex amplitude at each wavelength and then the three monochromatic images corresponding to each wavelength are reconstructed and combined into full-color images in the computer. Laser power variation over the wavelengths can be compensated for in the reconstruction process. We compared the images reconstructed by two algorithms using a single FFT and the double FFT methods with each other by both experiments and numerical simulations. Phase shift was correctly provided for the middle wavelength and errors arising at the other two wavelengths proved not to cause serious deterioration in the reconstructed images. We also employed an achromatic phase shifter consisting of polarization components to eliminate the errors. The optical system for this method, however, becomes more complicated.

![Figure 5-5. Setup for phase-shifting color digital holography](image)

2.5 Data compression

For storage or transmission of enormous amounts of frames comprising the complex amplitude which are necessary for dynamic objects or movies
we need to compress this information for the following purposes. The first is to record holograms at high frame rate and to increase memory capacity. The second is to accelerate data transmission through internets for reconstructing 3-d images at remote places. The third is the real time reconstruction by a spatial light modulator whose dynamic range is generally lower than that of CCD. We have recently studied the effects of reduction of bit-depths in hologram recording by phase-shifting digital holography and found that only four bits, that is, sixteen intensity levels are enough for reconstruction of satisfactory images. Here we describe the effects of information reduction of complex amplitude for image reconstruction by maintaining only phase. It has been known that phase of the object spectrum is essential for image formation of a scattering object. This property was first used for kinoforms that realized phase modulation corresponding to the computed Fourier spectrum of a virtual object by bleaching a photographic plate. Now for real objects the same data compression can be easily carried out by phase-shifting digital holography. It has been demonstrated by experiments that an image of a diffusely reflecting object can be reconstructed only by phase data of the derived complex amplitude.

The experimental results are shown in Fig.5-6. As expected we cannot reconstruct any image if we use only the amplitude distribution. It is seen that reduction of bit depth of the phase data does not seriously damage the image even down to 2 bits. We observed enhancement of halo in the image with low bit-depths. This halo results from ignorance of amplitude variation. This tendency was verified quantitatively by one-dimensional simulation where a diffusely reflecting object is represented by random phase variation. Smoothing of the images reconstructed from the compressed data has proved to be effective for enhancing image quality. Figure 5-7 represents the dependence of the root-mean-square difference of the reconstructed image intensity on the bit-depth. The caption of phase & amplitude means the quantization of both real and imaginary parts of complex amplitude. The residual difference in the phase-only data can be attributed the halo component. The quantized phase data can also be supplied to a spatial phase modulator, for example, using liquid crystal cells for real time reconstruction of 3-d images. If we use the spatial light modulator whose pixel pitch is equal to that of the CCD used for hologram recording, we could reconstruct the 3-dimensional image with unit magnifications both in the lateral and the axial directions and without any geometrical distortion.

Data compression for data transmission and encryption using phase-shifting digital holography were also discussed. The method used mainly was the reduction of bit-depth of the real and imaginary parts of complex amplitude.
Figure 5-6. Images reconstructed by 12 bit amplitude and phase (a), only 12 bit phase (b), only 12 bit amplitude (c), with 2 bits phase (d), and 1 bit phase (e).

Figure 5-7. RMS image difference between the image reconstructed from full complex amplitude and those from complex amplitude and phase only with reduced bit-depth.
3. MEASUREMENT OF SURFACE SHAPE AND DEFORMATION

3.1 General remarks and observed quantities

In holographic interferometry and electronic speckle pattern interferometry (ESPI) we can measure deformation of diffusely reflecting surfaces by superposing the complex amplitudes or the speckle intensities before and after surface deformation. Moreover, if we superpose the complex amplitudes before and after a change of object illumination, namely, that of the incident angle or the wavelength, we can obtain fringe patterns indicating surface contours. If the illumination is fixed and an object is deformed, the coherent superposition of the waves gives rise to interference or correlation fringes which mean contour lines of displacement along a direction that depends on the optical system. These fringe patterns can be automatically analyzed either by phase-shifting analysis or Fourier transform method\(^1\), both of which provide phase changes that are wrapped between 0 and \(2\pi\). After phase-unwrapping we obtain distributions of unwrapped phase which are proportional to displacement or surface height distributions. In digital holography, on the other hand, the wrapped phase values can be directly calculated from the difference of the reconstructed phases before and after the changes of illumination or object deformation. Hence we only need phase unwrapping to derive displacement or surface height. The optical setup for digital holography is also simpler because we essentially need no imaging lens and thus can analyze three-dimensional objects by means of numerical focusing. We describe below the quantities to be detected in these methods by applying a theory developed previously\(^2\).\(^3\).

In holographic interferometry applied to deformation measurement images of the object before and after deformation that is represented by the point displacement distribution \(a(r)\) are coherently superposed as shown in Fig.5-8. The effect of the lens aperture used for fringe observation is essential on the shape and contrast of fringe pattern that consists of speckle structure as shown in the figure. The mean size of speckle is given by the wavelength divided by the angular size of the lens aperture. We represent the observation point by the coordinate \((R,Z)\) where \(R\) is the transverse coordinate and \(Z\) is the distance from the conjugate plane of the object as shown in Fig.5-9. If the complex amplitudes at the point before and after object deformation are denoted by \(U_1(R,Z)\) and \(U_2(R,Z)\), the intensity averaged over an area much larger than speckle size but smaller than fringe spacing is given by
where the averaging is mathematically performed over a statistical ensemble of the microscopic structure of the object to discuss shape and contrast of fringe patterns for general object displacement and optical systems. This averaged intensity can be rewritten in terms of the unit vectors directed to the point source and along the direction of observation $I_s$ and $I_o$ in such a form as

$$\langle I(R, Z) \rangle = (\langle U_1(R, Z) + U_2(R, Z) \rangle)^2 = \langle J_1(R, Z) \rangle + \langle J_2(R, Z) \rangle + 2\Re(U_1(R, Z)U_2^*(R, Z))$$

(5-12)
where we have assumed $<I_0>=<I_1>=<I_2>$. This equation represents superposition of interference intensity between identical points before and after object deformation. The fringe pattern means the contour lines of the displacement component along the bisector between the direction of the illumination source and that of observation. The fringe pattern has the complex contrast that represents the fringe contrast and phase and is given by

$$\gamma \exp(i\alpha) = \langle U_1(R;Z)U_2^*(R;Z) \rangle / \langle I_0(R;Z) \rangle$$

that depends on the ratio of speckle displacement caused by object deformation to the mean speckle size as illustrated schematically in Fig.5-8. If the speckle displacement is smaller than the mean size, the contrast of the fringes is high. On the other hand, if the speckle displacement exceeds the mean size, the contrast vanishes. This is because the phase relationship is random between difference speckles. The speckle displacement depends on object deformation and the position of the observation plane in a complicated manner, where the mean speckle size is proportional to the distance of the observation plane from the lens aperture and inversely proportional to the size of observation aperture. Since speckle displacement does not depend on the aperture size, we can increase the fringe contrast by reducing the aperture. However, if the speckle size approaches the fringe spacing that depends on slope of displacement corresponding to rotation and strain, the above discussion on the fringe contrast based on the average intensity will cease to be valid. Anyway digital holography enables us to analyze phase distributions of scattered light quantitatively and hence this situation might be investigated in more detail.

In electronic speckle pattern interferometry (ESPI) speckle patterns formed by interferometric setups which combine at least one reflected by the object are recorded by a CCD camera, subtracted from the initial pattern, and squared by video circuits or a computer to display contour lines of object displacement. The displayed brightness is proportional to

$$V_S = \left( \langle I_{S_1} - I_{S_2} \rangle \right)^2 = \left( \langle I_{S_1}^2 \rangle - \langle I_{S_2}^2 \rangle - 2\langle I_{S_1}I_{S_2} \rangle \right),$$

where the third term on the right-hand side represents the correlation fringes.

In an arrangement that is used for measurement of out-of-plane displacement (Fig.5-10) the intensities recorded by the CCD camera are given by
\[ I_{S1} = \left| U_1 + U_R \right|^2, \quad I_{S2} = \left| U_2 + U_R \right|^2 \] (5-16)

with the object amplitudes \( U_1 \) and \( U_2 \) at the image plane \( Z=0 \) and the complex amplitude \( U_R \) of the reference light reflected from a fixed rough surface. The averaging contained in Eq.(5-15) is physically carried out by the low-pass filtering introduced electronically and/or visually. Substitution of Eq.(5-16) into Eq.(5-15) yields

\[ V_S = 2 \left( \langle I_o \rangle^2 + 2 \langle I_o \rangle \langle I_R \rangle - 2 \langle I_R \rangle \Re \langle U_1 U_2^* \rangle - |\langle U_1 U_2^* \rangle|^2 \right) \] (5-17)

where we considered the statistical independence between the reference amplitude \( U_R \) and the object amplitude \( U_1, U_2 \). The third term in the right-hand side of this equation represents the fringe pattern that is the same as that derived in Eq.(5-12) for holographic interferometry where we set \( Z=0 \).

In digital holography we can directly evaluate values of phases as \( \arg(U_1) \) and \( \arg(U_2) \) whose difference becomes

\[ \Phi = \langle \arg(U_1) - \arg(U_2) \rangle = \frac{\langle U_1 U_2^* \rangle}{\sqrt{\langle I_1 \rangle \langle I_2 \rangle}} = -(k_s - k_o) \cdot a + \alpha, \] (5-18)
where \( k_s = -k_l \) and \( k_o = k_o \) mean the wave vectors of illumination and reflected light. The phase difference given by Eq.(5-18) cannot be determined when \( I_1 \) or \( I_2 \) vanish corresponding to dark speckles. Now we also consider the interference between the identical points for studying the sensitivity of the contouring and deformation measurement. In the following we derive the phase change corresponding to Eq.(5-18) in surface contouring using change of object illumination.

### 3.2 Surface contouring

For surface shape measurement we calculate the phase difference due to change of illumination, that is, changes of the incident angle or wavelength. The wave vectors of illumination beam are represented by \( k_a \) and \( k_b \) as shown in Fig.5-11. If we denote the vectors representing the observation direction by \( k_{ao} \) and \( k_{bo} \), the difference of the reconstructed phase corresponding to the identical points is given by

\[
\Phi(x,y) = -(k_{az} - k_{bz} - k_{azo} + k_{bzo}) K(x,y) - (k_{ax} - k_{bx}) x
\]

(5-19)

where we have assumed the incident plane to be included in the x-z plane and the reference plane for the surface height to be the x-y plane. The first term on the right hand side of Eq.(5-19) means the phase difference proportional to the surface height and the second term stands for tilt components.

If we change the wavelength of the illumination from \( \lambda_a \) to \( \lambda_b \) with the incident angle \( \theta_i \) and normal observation and reconstruct each of the hologram with the same wavelength as in hologram recording, the difference of the phases which are reconstructed with the same wavelength as in the recording is expressed by

\[
\Phi(x,y) = -(1 + \cos \theta_s) (k_a - k_b) h(x,y) = -2 \pi (1 + \cos \theta_s) h(x,y) / \Lambda, \quad (5-20)
\]

which means the contours of object height with a sensitivity that is associated with the synthetic wavelength defined by

\[
\Lambda = 1 / |1 / \lambda_a - 1 / \lambda_b| \quad (5-21)
\]

When the incident angle is changed from \( \theta \) to \( \theta + \Delta \theta \), the phase difference becomes
\[
\Phi(x,y) = -k[\cos \theta - \cos(\theta + \Delta \theta)]h(x,y) - k[\sin \theta - \sin(\theta + \Delta \theta)]x
\]
\[
= 2kh(x,y)\sin\left(\theta + \frac{\Delta \theta}{2}\right)\sin\frac{\Delta \theta}{2} + 2kx \sin\frac{\Delta \theta}{2} \cos\left(\theta + \frac{\Delta \theta}{2}\right)
\]
\[
\approx kh(x,y)\Delta \theta \sin \theta + kx\Delta \theta \cos \theta
\]

(5-22)

where the bottom line results from an approximation for \(\Delta \theta\) to be much smaller than unity. This phase distribution is just equal to the phase of the projected fringes produced by the two coexisting beams. For small \(\Delta \theta\) the height sensitivity of the phase difference is given by \(\lambda/\Delta \theta \sin \theta\).

An actual setup used in experiments\textsuperscript{22} is shown in Fig.5-12. The collimated laser beam is incident on a plane mirror that is rotated by a stepping motor under computer control. Three phase-shifted holograms were recorded before and after the rotation of the mirror. The resultant distributions of the reconstructed phase before and after the mirror rotation are subtracted from each other to produce the unwrapped phase. After phase-unwrapping we obtain phase distribution that is given by Eq.(5-22) from which the tilt component has to be subtracted to provide surface height distribution with respect to the reference plane. The presence of the tilt component adds to the fringe density to disturb phase-unwrapping more seriously than in its absence resulting from the dual wavelength method. Any way phase-shifting digital holography improves accuracy of surface contouring and deformation measurement than the off-axis method because of more pixels used for the reconstructed images.
Figure 5-12. Arrangement for surface shape measurement

Figure 5-13 represents the result obtained from the dual incident angle method applied to a miniature bulb which was painted white. The distributions of the phase difference shown in (a) contain the carrier component corresponding to the second term of the right-hand side of Eq.(5-22). We subtract this component from the phase difference at 1,024x1,024 pixels before phase-unwrapping. This procedure aligns the reference plane parallel to the object plane as shown in Fig. 5-13 (b). The resultant distribution contains noise associated with speckles. We suppressed this noise by extracting one point from each 2x2 matrix where the modulus of the product $|U_1U_2^*|$ becomes maximal. This filtering is based on the fact that the phase value is more reliable for higher amplitude. The compressed data are then smoothed by averaging over each 2x2 matrix with final data pixels of 256x256. Figure 5-14 (a) and (b) represent the 3-d maps and the cross-sections through the bulb axis before (a) and after the averaging (b). This method of surface shape measurement could be easily applied to small objects by employing a microscope system mentioned above.

In the dual wavelength method we can employ the normal incidence that is free from the shadowing effect as well as from correction of the reference plane. The wavelength shift is most easily realized by changing the injection current of a laser diode. We can use a mode hop that provides the shift of a several tenths of nanometers. The synthetic wavelength is given by $\Lambda=\lambda^2/\Delta\lambda$ where we assume that the wavelength shift is much larger than the initial wavelength. This method was also used in conventional holography where reconstruction is performed with the same wavelength to introduce the chromatic aberration into the image recorded at the shifted wavelength. In digital holography is free from this aberration. Phase-shifting digital
holography is especially suited because of lack in the fringe carrier that depends on recording wavelength.

**Figure 5-13.** Phase difference before (a) and after (b) removal of tilt.

**Figure 5-14.** 3d-maps and cross-sections before (a) and after (b) nonlinear filtering.
3.3 Deformation measurement

For deformation measurement the illumination is fixed and the phase difference of the reconstructed waves before and after object deformation depends on the vectors illustrated in Fig. 5-15. If the speckle displacement is much smaller than the mean size, the phase difference becomes

\[ \Phi(x,y) = -(k_s - k_o) \cdot a(x,y) \] (5-23)

This condition means that the systematic phase change can be observed as a result of cancellation of random phase variation between different speckles. It limits the measurement of in-plane deformation as illustrated in Fig. 5-8.

![Figure 5-15. Principles of deformation measurement by digital holography](image)

In order to measure both surface shape and deformation of diffusely reflecting surfaces we built the optical system shown in Fig. 5-16. We employed a setup for image hologram so that limitation on the object size can be relaxed while ensuring high light flux incident on the CCD\(^2\). Surface shape is measured by tilting a mirror for object illumination. This setup is the same as that of phase-shifting electronic speckle interferometry, but it can also be used for three-dimensional object by virtue of numerical focusing. We need not refocus the imaging lens on the position of interest and have only to record the hologram once before and after object deformation or mirror tilt. We also have much more freedom for suppression of speckle noise because phase difference is directly derived instead of fringe intensity detected in ESPI as mentioned above. Speckle displacement can be detected from the cross-correlation peak of the reconstructed intensities.
Figure 5-16. Setup of imaging digital holography for measurement of surface shape and deformation

Figure 5-17 shows the result of experiments conducted for out-of-plane deformation of a square plate of aluminum with the size of 50x50 mm$^2$ and 1 mm thick. It was pushed at the center while being supported at the circular edge. The phase difference shown in Fig.5-17 (a) is unwrapped to deliver the distribution of out-of-plane displacement. The distribution of the displacement along the line indicated is displayed in Fig.5-17 (b). The resolution is estimated to be 0.01 μm. The surface shape after total loading repeated 20 times is represented by Fig.5-18, where (a) is the phase difference arising from illumination tilt of $\Delta\theta=0.25$ degrees from $\theta_s=45$ degrees, (b) after removal of carrier component, and (c) is the cross-section along the indicated line. The fluctuation is about 10 μm that is an order of surface roughness.

If we use the two wavelength contouring, the setup becomes even simpler what would be very important for industrial applications. A unique capability of digital holography to record amplitude and phase simultaneously and to analyze them separately will provide new tools for shape and deformation measurements where digital speckle correlation technique for detecting in-plane displacement$^{25,26}$ can also be involved. This point will be extremely important because all the coherent-optical techniques consisting of holographic interferometry and speckle methods will be merged to digital holography that can treat three-dimensional information automatically and quantitatively with simple optical setups. Transmission of holographic data through internet and 3-dimensional reconstruction at remote locations is another incomparable feature of digital holography. Comparison with Finite Element Analysis will thus become easier.
3.4 Vibration analysis

Digital holography can also be applied to continuous deformation such as vibration. We used the setup shown in Fig. 5-19 where single-mode optical fibers are used for guiding light from a single-mode He-Ne laser. We record time-averaged hologram intensity by keeping the vibration frequency much
higher than the frame rate of CCD that is equal to 30 Hz. Three time-averaged holograms are also recorded by shifting the reference phase. The object amplitude is given by

\[ U_O(x', y', t) = U_E(x', y') \exp \left\{ i \left[ \alpha(x, y) \cos(\Omega t + \phi(x, y)) \right] \right\} \]  

(5-24)

where \( U_E \) means the object amplitude representing the neutral position of vibration and \( \Omega \) and \( \phi \) denote angular frequency and phase of vibration, respectively. The resultant time-averaged intensity to be recorded by a CCD is given by

\[ S_H(x, y; \delta) = \left[ I_H(x, y; \delta) dt = \int |U_R(x, y, t)|^2 dt + 2 \Re \left[ \int U_R(x, y, t) U^*(x, y, t) dt \exp(i\delta) \right] \right] \]

(5-25)

This is substituted into Eq.(5-24) to derive time averaged object complex amplitude that is to be Fresnel transformed. The reconstructed intensity becomes

\[ U_I(x, y, -z_O) = U_{OA}(x, y) = U_E(x', y') J_0[ka(x, y)] \]

(5-26)

where \( J_0 \) means the 1st-kind Bessel function of the 1st order. This relationship suggests that we can observe the same fringe pattern as in conventional holography using photographic recording. The experimental intensity distributions obtained from a buzzer of 40 mm in diameter and vibrating at a resonant frequency of 5 kHz are displayed in Fig.5-20. We see that the pattern (a) resulting from the present method occupies the whole pixel number of the used CCD and is free from other additional components. However, the image (b) obtained from the off-axis setup and the same CCD contains the zero-th and the conjugate image together and occupies only one-nine-th of the whole pixels. Therefore, phase-shifting digital holography permits us to employ larger objects and provides much better image quality to improve the measurement accuracy and spatial resolution.

If we add AC phase modulation to the reference arm at the vibration frequency with a phase-modulator consisting of a PZT cylinder on which a single-mode fiber is wrapped, we can shift the position of the highest intensity from zero amplitude to an arbitrary amplitude corresponding to that of phase modulation\(^{27}\). Thus we can extend the range of measurable amplitude. Although the reconstructed phase takes the values of either 0 or \( \pi \), the distribution of phase might be utilized for identifying the positions of vibration nodes and antinodes with higher accuracy.
3.5 S/N ratio of the phase differences and its improvement

Accuracy of shape and deformation measurement depends on the dispersion of phase differences that is described by correlation properties of speckle. It is mostly affected by in-plane object displacement. At the conjugate plane of the object the speckle displacement is equal to in-plane displacement times imaging magnification, while at the defocused plane the gradient of Eq.(5-19) times an amount of defocusing adds to the above component. The plane where the speckle displacement becomes minimum correspond to the plane of fringe localization in holographic interferometry. In conventional holographic interferometry it was difficult to compensate for
the effect of in-plane displacement. It was only possible by real time interferometry where hologram position is adjusted and by a sandwich hologram\textsuperscript{28} where the mutual position of the layered holograms was shifted. In digital holography, however, these mechanical adjustments are got rid of and mutual displacement between the reconstructed phase distributions can be delivered automatically and precisely from the peak position of the cross-correlations of phase or intensity distributions. In this way correlation between the laterally shifted wavefronts can be easily restored.

In the case of surface contouring there is always no lateral shift of the resultant wavefronts at the focused plane. At the defocused plane, however, the speckle displacement occurs owing to the change of illumination\textsuperscript{29}. This situation indicates that surface contouring of 3-dimensional object is limited by the speckle displacement at the defocused plane. In digital holography, however, this speckle displacement can be reduced to be zero by numerical refocusing at the desired plane. The refocusing also eliminates image blurring caused by defocusing. For these reasons image contouring by digital holography provides us with higher performance and larger measurement depth than the fringe projection method. The speckle noise due to random change of the amplitude can also be suppressed if we exclude the phase differences at the pixels where intensity is too low as mentioned in the experiments above. Since the procedure for deriving object phases is nonlinear with respect to intensity, a new method of speckle noise suppression might be expected that has been insufficient with linear averaging.

4. CONCLUSIONS

In this chapter we surveyed the principle and applications of phase-shifting digital holography that has raised performances of digital holography substantially. Since it delivers the values of complex amplitude at the CCD plane directly without any filtering process as required for the off-axis setup, both derivation and processing of information required for transmission, display, and measurement are straightforward and simple. The full number of CCD pixels is utilized for the entire image to provide image quality much better than in the off-axis setup. We discussed the image formation and applications to measurement of surface shape and deformation of diffusely reflecting objects.

In the section of image formation 3-dimensional microscopy has been explained that can applied for both amplitude and phase objects. Color holography using a laser emitting three-wavelength simultaneously and a
color CCD enables quick hologram recording and reconstruction with a satisfactory color balance.

For increasing memory capacity and transmission rate to remote places the derived complex amplitude can be compressed into phase-only data with a few bits without serious deterioration of image quality in the case of diffusely reflecting objects.

In measurements of surface shape and deformation the quantities to be evaluated in phase-shifting digital holography has been explained in comparison with those observed in conventional holographic interferometry and electronic speckle pattern interferometry (ESPI). We could say that digital holography, especially, phase-shifting digital holography has unified not only these methods but also will comprise speckle correlation technique together that detects speckle displacement and/or decorrelation due to surface deformation or illumination change.

The most serious issue in phase-shifting digital holography will be the recording time required for at least three phase-shifted patterns. In the experiments reported above it was 1 s with 1,024x1,024 pixels with 12 bits, while the reconstruction took a few seconds. The recording time could be shortened to less than 0.03 s by using a CCD with the frame rate of 120 per second for measuring the shape of liquid surface under convection. The limitation in recording time that needs at least three frames will be less serious when a high speed CCD is used. The simplicity in the reconstruction process also eases introduction of data compression for storage, transmission, and display of 3-d information based on 2-dimensional array of complex amplitude.

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