MOTION PATTERN SINGULARITY IN LOWER MOBILITY PARALLEL MANIPULATORS

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Abstract  Many procedures to detect singularities in manipulators have been described in the literature up to now. Singularities are often defined as an instantaneous or permanent modification in the number of degrees of freedom (DOF), either affecting certain links or the whole mechanism. However, the motion of the end-effector of a parallel manipulator is not only given by the number of DOF but also by the nature of them (rotational or translational). There are poses in which, being no quantitative alteration of the DOF of the platform, there are changes in this nature. This is also a singularity, and produces, as other singularities do, a mathematical deficiency in the velocity equations of the manipulator. This type of singularity affects only the so called lower mobility parallel manipulators. In this contribution the authors define this new type of singularity, called motion pattern singularity, and present a procedure to analyze it.

Keywords: Motion Pattern, Parallel Manipulators, Platform Twist, Singularities

1. Introduction

In robot design, knowing the possibilities of motion of the end-effector is the key to a proper choice of the application. The motion possibilities of the parallel manipulator’s end-effector are not only determined by the number of DOF, but also by the nature of these freedoms. This nature has a qualitative aspect (translational or rotational) and a quantitative one: the directions of possible translation or rotation at each pose. Both features define the motion pattern of the platform. This denomination has been previously used for parallel manipulators in Kong and Gosselin, 2005. The motion pattern is specially important in lower mobility parallel manipulators because it determines the possible task to be performed. In addition, a proper design looks for constant directions of the DOF inside the workspace. Unfortunately, in many of the lower mobility parallel manipulators there is a variation of the translational and rotational directions with the pose. This does not imply a singularity,
but it does condition the kinematic characteristics of the manipulator such as its manipulability.

The nature of the DOF of the moving platform is usually unaltered by the motion of the manipulator, the same with the number of DOF. Nevertheless, it is possible that, in certain poses of the platform, some of the DOF change in nature, e.g. some rotational DOF becomes translational. Obviously, this alters substantially the motion pattern of the platform and hence, it can be considered as a singularity. In fact, it generates a mathematical deficiency as it will be explained later. In these circumstances, it is possible that the robot were unable to accomplish with the intended task, at least instantaneously at that pose. It must be highlighted that such a condition is independent of the coexistence with any other type of singularity. Anyway, it is necessary to have a procedure to detect those variations in the nature of the DOF of the platform along its motion.

A wide bibliography on singularities in robots has been issued in the past. We bring notice to some well known references as Freudenstein, 1962, Hunt, 1978, Sugimoto et al., 1982, Merlet, 1989, Gosselin and Angeles, 1990, Zlatanov et al., 1994, and Park and Kim, 1999. These works and some others have stated fundamental concepts as direct kinematic singularity, inverse kinematic singularity, or increased mobility configuration. Some more specific concepts have also been issued, such as constraint singularity Zlatanov et al., 2002, architecture singularity Ma and Angeles, 1992, internal singularities Company et al., 2006 or cuspidal manipulator Wenger, 2004. In all these works the singularity is understood as an alteration in the number of DOF, either globally in the mechanism as a whole, or locally at some part of it (preferably at input or output). However, to our best knowledge, such singularity as the one described in this paper has not been discussed. In fact, the latest references on parallel manipulator singularities as Thomas et al., 2005, Huang and Cao, 2005, or Liu et al., 2005 go over the quantitative modification of the number of DOF again.

2. Motion Pattern Singularity

In this section the authors present the definition and procedure to obtain the motion pattern of the manipulator’s platform as well as the singularity associated to that concept.

2.1 Motion Pattern

The motion pattern of a parallel manipulator represents the platform’s capacity of motion. The number of degrees of freedom, their
nature (translational or rotational) and directions define this characteristic. Although this is an instantaneous feature, the number and nature of the DOF are, generally, permanent in the workspace. In fact, when they change in some pose, it is because it is a singular configuration. Against, the directions of translation and rotation of the platform’s DOF are often variable in parallel manipulators.

The motion pattern is obtained with a procedure that starts with a velocity equation that maps joint velocities to the platform’s twist:

\[ \dot{x} = J \dot{q} \]  

(1)

where \( \dot{x} = [\dot{p}^T \omega^T]^T \) is the twist of the moving platform, being \( \dot{p} \) the velocity of a point \( P \) in the platform and \( \omega \) the platform’s angular velocity, \( J \) is a Jacobian matrix and \( \dot{q} \) is the vector of input joint rates. This Jacobian matrix is not always easy to find analytically. It will be often found numerically, and in those occasions the motion pattern is analyzed pose by pose.

This Jacobian can be divided into two submatrices \( J_T \) and \( J_R \), corresponding to terms that affect linear and angular velocity respectively. If the full cycle mobility, Hunt, 1978, of a non-redundant manipulator is \( F \) Eq. 1 is

\[ \begin{bmatrix} p \\ \omega \end{bmatrix}_{6 \times 1} = \begin{bmatrix} J_T \\ J_R \end{bmatrix}_{6 \times F} \dot{q}_{F \times 1} \]  

(2)

The rotational motion space is analyzed extracting the equations corresponding to angular components from that system:

\[ \omega = J_R \dot{q} \]  

(3)

The number of rotational DOF of the platform \( F_R \) is the rank of matrix \( J_R \) and the corresponding directions \( \omega_r \) are obtained in Eq. 3 with a basis \( \dot{q}_r \) of the rangespace of that matrix:

\[ \omega_r = J_R \dot{q}_r \quad r = 1 \ldots F_R \]  

(4)

The number of translational DOF of the platform \( F_T \) is \((\text{rank}(J) - F_R)\), and the corresponding directions \( \dot{p}_t \) are solved in the translational part of Eq. 2 upon substitution of a basis \( \dot{q}_t \) of the null space of matrix \( J_R \):

\[ \dot{p}_t = J_T \dot{q}_t \quad t = 1 \ldots F_T \]  

(5)

Therefore, the platform’s motion pattern is defined by the number of rotational DOF, \( F_R = \text{rank}(J_R) \), and translational DOF, \( F_T = \text{rank}(J) - \text{rank}(J_R) \), along with the rotational and translational directions, \( \omega_r \) and \( \dot{p}_t \) respectively.
2.2 Singularity

A motion pattern singularity occurs in a pose where, being no alteration in the number of DOF of the moving platform, some DOF change in nature, e.g. a rotational DOF becomes translational or vice versa. Note that this singularity does not make reference to variations in the directions of possible translation or rotation of the platform. In fact, this latter is quite usual in lower mobility parallel manipulators although constant directions are desirable in design. Obviously, this type of singularity is only possible in lower mobility parallel manipulators.

Other type of singularities affecting the moving platform, and already described in the references, are the constraint singularity defined in Zlatanov et al., 2002 and the so called Impossible Output introduced in Zlatanov et al., 1994. The former implies a gain while the latter means a loss in the DOF of the end-effector.

The rank of Jacobian $J_R$ has to be checked to detect mathematically the motion pattern singularity. As this Jacobian is homogeneous in terms of units, the first singular value not null of that Jacobian serves well as an indicator of closeness to singularity. The Singular Value Decomposition technique applied to $J_R$ provides both the range and null spaces, and the motion pattern of the platform at the singularity is obtained with them.

3. Example

A 3 DOF parallel manipulator with rotational motion is shown in Fig. 1. A passive limb with 3 revolute joints (R) constrains the desired motion while three linear actuators (SPS) provide the control of the end-effector. A fixed frame is defined with origin at point $O$ and a moving frame is attached to the platform with origin at point $P$. The loop-closure position equation is stated for every limb relating vector $p$ that positions point $P$ with: vectors $a_i$ that locate the fixed S joints $A_i$ of the linear actuators, vectors that go from $A_i$ to $B_i$ (being $s_i$ a unit vector in the direction of actuators and $\rho_i$ the length of the actuator), and vectors $p_{bi}$ that place points $B_i$ with respect to point $P$ in the platform and are best expressed in the moving frame:

$$ p = a_i + \rho_is_i - p_{bi} \quad i = 1, 2, 3 $$

Differentiating Eq. 6 with respect to time yields

$$ \dot{p} = \dot{\rho}_i \cdot s_i + \omega_i \times \rho_i s_i - \omega \times p_{bi} \quad i = 1, 2, 3 $$

where $\dot{\rho}_i$ is the actuator’s rate, $\omega_i$ are the angular velocities of the actuators, and $\omega$ is the angular velocity of the platform. If now we dot
multiply Eq. 7 by \( s_i \) and simplify wherever possible we get

\[
\dot{s}_i \cdot \dot{p} + [p_i \times s_i] \cdot \omega = \dot{r}_i \quad i = 1, 2, 3 \tag{8}
\]

The revolute joints of the passive limb constrain the platform to a 3 DOF rotational motion. Let’s consider the geometrical constraints imposed by each of the revolute joints in the passive limb. These can be formulated taking into account the fixed orientation of the position vector \( p \) with respect to each of the R joints’ axes \( r_i \), namely

\[
\begin{align*}
    r_1 \cdot p &= 0 \tag{9} \\
    r_2 \cdot p &= p \tag{10} \\
    r_3 \cdot p &= 0 \tag{11}
\end{align*}
\]

Differentiating Eq. 9 to 11 with respect to time yields

\[
\begin{align*}
    r_1 \cdot \ddot{p} &= 0 \tag{12} \\
    r_2 \cdot \ddot{p} &= 0 \tag{13} \\
    r_3 \cdot \ddot{p} + [r_3 \times p] \cdot \omega &= 0 \tag{14}
\end{align*}
\]

Compiling Eq. 8 and Eqs. 12 to 14 in matrix form gives

\[
\begin{bmatrix}
    r_1^T & 0^T \\
    r_2^T & 0^T \\
    r_3^T & [r_3 \times p]^T \\
    s_1^T & [p_i \times s_i]^T \\
    s_2^T & [p_i \times s_i]^T \\
    s_3^T & [p_i \times s_i]^T
\end{bmatrix}
\begin{bmatrix}
    \dot{p} \\
    \omega
\end{bmatrix} =
\begin{bmatrix}
    0 \\
    I
\end{bmatrix}
\dot{\rho} \tag{15}
\]
This is a velocity equation that maps the twist of the platform to the inputs of the manipulator. A rank deficiency in the first Jacobian in Eq. 15, called $J_x$, imply a direct kinematic singularity. A rank deficiency of the second Jacobian, called $J_q$, is not possible in this manipulator, and hence no singularity in the inverse problem exists.

The Jacobian $J_x$ must be inverted and postmultiplied by the second Jacobian to get the following expression of the twist

$$\begin{bmatrix}
\dot{p} \\
\dot{\omega}
\end{bmatrix} = \frac{1}{|J_x|} \begin{bmatrix}
\delta_1 & \delta_2 & \delta_3 \\
\alpha_1 & \alpha_2 & \alpha_3
\end{bmatrix} \dot{\rho} = \begin{bmatrix}
J_T \\
J_R
\end{bmatrix} \dot{\rho}$$

(16)

where the translational Jacobian $J_T$ is formed by vectors $\delta_i$ multiplied by $\frac{1}{|J_x|}$:

$$\delta_1 = [n_3 \cdot (m_3 \times m_2)] \cdot (r_1 \times r_2)$$

(17)

$$\delta_2 = [n_3 \cdot (m_1 \times m_3)] \cdot (r_1 \times r_2)$$

(18)

$$\delta_3 = [n_3 \cdot (m_2 \times m_1)] \cdot (r_1 \times r_2)$$

(19)

being

$$n_3 = r_3 \times p$$

(20)

$$m_1 = p b_1 \times s_1$$

(21)

$$m_2 = p b_2 \times s_2$$

(22)

$$m_3 = p b_3 \times s_3$$

(23)

And the rotational Jacobian $J_R$ is formed by vectors $\alpha_i$ multiplied by $\frac{1}{|J_x|}$:

$$\alpha_1 = -[r_3 \cdot (r_1 \times r_2)] \cdot (m_3 \times m_2) - [s_2 \cdot (r_1 \times r_2)] \cdot (n_3 \times m_3)$$

$$- [s_3 \cdot (r_1 \times r_2)] \cdot (m_2 \times n_3)$$

(24)

$$\alpha_2 = -[r_3 \cdot (r_1 \times r_2)] \cdot (m_1 \times m_3) - [s_1 \cdot (r_1 \times r_2)] \cdot (m_3 \times n_3)$$

$$- [s_3 \cdot (r_1 \times r_2)] \cdot (n_3 \times m_1)$$

(25)

$$\alpha_3 = -[r_3 \cdot (r_1 \times r_2)] \cdot (m_2 \times m_1) - [s_1 \cdot (r_1 \times r_2)] \cdot (n_3 \times m_2)$$

$$- [s_2 \cdot (r_1 \times r_2)] \cdot (m_1 \times n_3)$$

(26)

Then, the procedure to analyze the motion pattern can be applied and possible singularities in the motion pattern found.

In a nonsingular position the rangespace of the rotational Jacobian $J_R$ has a dimension of 3, and hence the platform has 3 rotational DOF. The rank of $J$ is also 3, and the nullspace of $J_R$ has a zero dimension, therefore the platform has 0 translational DOF.
However, in any pose where the revolute axis $r_3$ is parallel to the joint axis $r_1$ (see Fig. 2) the rank of the rotational Jacobian $J_R$ decreases. Vectors $\alpha_i$ are

\[
\alpha_1 = -[s_2 \cdot (r_1 \times r_2)] \cdot (n_3 \times m_3) - [s_3 \cdot (r_1 \times r_2)] \cdot (m_2 \times n_3) \tag{27}
\]

\[
\alpha_2 = -[s_1 \cdot (r_1 \times r_2)] \cdot (m_3 \times n_3) - [s_3 \cdot (r_1 \times r_2)] \cdot (n_3 \times m_1) \tag{28}
\]

\[
\alpha_3 = -[s_1 \cdot (r_1 \times r_2)] \cdot (n_3 \times m_2) - [s_2 \cdot (r_1 \times r_2)] \cdot (m_1 \times n_3) \tag{29}
\]

Note that the three of them are perpendicular to $n_3$ and hence dependent. The dimension of the rangespace is 2, the possible rotation has 2 DOF and its direction is on the plane perpendicular to $n_3$.

The nullspace is one dimensional, and upon substitution into the translational Jacobian $J_T$ the direction of translation is obtained. In view of Eqs. 17 to 19 is easy to note that every vector $\delta_i$ that form the translational Jacobian $J_T$ is parallel to vector $r_1 \times r_2$. Therefore this is the only direction of possible translation. Adequate inputs can produce a finite motion with this motion pattern.

![Figure 2. Sequence of motion in a Motion Pattern Singularity ($r_1 = r_3$).](image)

### 4. Conclusions

This paper describes a singularity that affects the moving platform of lower mobility parallel manipulators. As far as the authors know, this type of singularity has not been defined in the references on this subject. This singularity does not cause an increment or reduction of the DOF of the end-effector, but a change in their nature (transforming from rotational to translational or vice versa). Regarding the manipulator performance, this singularity may produce problems with the control or the actuators. If the manipulator approaches a pose where there is a motion pattern singularity, it is evident that the desired twist in the end-effector may require very high inputs. In practice this produces abrupt increments of the inputs that can damage the machine, or simply not move the platform at all. As with other types of singularities, the
motion pattern singularity is generally instantaneous, i.e. dependent on the pose. However, there are many cases where the singularity becomes permanent for certain inputs.

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