A MATLAB-Based Graphical Technique for Amortization Study of Adjustable Speed Drives

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Abstract—Economic justification of electric drives is the primary reason for many applications. However, the cumbersome trial-and-error calculations necessary to present this aspect often leads to omission of the topic in electric drives courses. This paper presents a MATLAB-based program that generates plots of input power and shaft torque for variable-frequency operation of three-phase induction motors. Based on these graphs, an example is presented illustrating how a cost amortization study for an adjustable speed drive application can be presented with minimal effort and lecture time.

Index Terms—Electric drives, motor-drive cost study, variable-frequency induction motors.

I. INTRODUCTION

A N INTRODUCTORY course in electric drives is a common offering to upper level electrical engineering undergraduate students and first-year graduate students. Typically, significant time is dedicated to torque-speed analysis methods for the all-important inverter-fed induction-motor adjustable speed drive (ASD). However, little, if any, time is spent in quantitatively addressing the resulting efficiency of this drive and the cost-saving potential that it offers in many applications. An underlying reason for this omission is the cumbersome calculations required for such study. This paper introduces a graphical technique that allows inclusion of economic study in an electric drive course while increasing the understanding of an ASD performance nature. The necessary graphs are generated by MATLAB analysis. Once the procedures of analysis are understood, the mundane portion of the work can be relegated to the computer, allowing the student to conduct an economic study with minimum computational effort while gaining practical knowledge for potential use in industrial practice.

II. THEORETICAL BASIS

Since computer analysis will be performed to generate graphs, there is no reason to sacrifice accuracy by use of an approximate equivalent circuit as commonly introduced to simplify computation by handheld methods [1]–[3]. The exact per phase equivalent circuit for analysis is shown by Fig. 1, where the frequency dependence of reactances is explicitly shown. Further, the fact that the core-loss resistor value is a function of frequency will remain near rated value for all points of analysis so that variation of \( R_c \) and \( L_m \) with flux density need not be considered.

A. Equivalent Circuit Values

With knowledge of the rated frequency (\( f_R \)) and rated voltage (\( V_{LR} \)) equivalent-circuit reactance values, the leakage and magnetizing reactances follow as [4]

\[
L_{\Omega 1} = \frac{X_1}{2\pi f_R}, \quad L_{\Omega 2} = \frac{X_2}{2\pi f_R}, \quad L_m = \frac{X_m}{2\pi f_R},
\]

(1)

Assuming that harmonic winding currents are negligibly small, the total hysteresis and eddy current losses (\( P_c \)) modeled by the core-loss resistor (\( R_c \)) are described by [5]

\[
P_c = K_c B_m^3 f^2 + K_h B_m^2 f \quad (2)
\]

where \( B_m \) is the peak value of air-gap flux density. The per phase core-loss resistance value is determined by [4]

\[
R_c = \frac{P_c/3}{E^2} = \frac{P_c/3}{\omega^2 B_m^2/2} \quad (3)
\]

Let \( a \) be the per unit portion of \( P_c \) attributable to eddy currents within the magnetic core. Since \( B_m \) is to be held constant by the adopted V/Hz control, (2) and (3) can be used to predict the value of the core-loss resistance (\( R_c(f) \)) for any frequency (\( f \)) based on the value of core-loss resistance (\( R_c \)) known from a standard rated frequency, no-load test of the induction motor

\[
R_c(f) = \frac{a f_R}{a f^2 + (1-a) f_R} R_c \quad (4)
\]

Unless specific information on segregation of eddy current and hysteresis losses is known for the motor ferromagnetic material, use of \( a = 0.5 \) is suggested.

B. Characterization of Windage and Load

Usually a reasonably good approximation to the motor friction and windage torque (\( T_{FW} \)) and the load torque (\( T_L \)) at any...
speed \((\eta_m)\) can be obtained by appropriate choice of constants \((a_0, \ a_1, \ b_0, \ b_1, \ k_F, \ k_p)\) for the equations
\[
T_{FW} = a_0 + a_1 (\eta_m)^{k_F} \tag{5}
\]
\[
T_L = b_0 + b_1 (\eta_m)^{k_F}. \tag{6}
\]

C. Motor Shaft Torque and Input Power

Based on Fig. 1 and polyphase induction motor theory from an introductory course in electric machines \([4]–[7]\), the per phase input impedance \((Z_{in})\) for any shaft speed \((\omega_m)\), and impressed line voltage \((V_L)\) at frequency \((f)\) can be calculated for a \(p\)-pole motor. Let
\[
Z_{11} = R_1 + j2\pi f L_{11} \tag{7}
\]
\[
Z_{22} = \frac{R_2'}{s} + j2\pi f L_{22} \tag{8}
\]
\[
Z_m = j2\pi f L_m R_c(f)/[R_c(f) + j2\pi f L_m] \tag{9}
\]
where synchronous speed \((\omega_s)\) and slip \((s)\) are defined by
\[
\omega_s = \frac{2}{p} \omega = \frac{4\pi f}{p} \quad \text{and} \quad s = \frac{\omega_s - \omega_m}{\omega_s}, \tag{10}
\]
Then, the input impedance is
\[
Z_{in} = Z_{11} + Z_{22} Z_m/(Z_{22} + Z_m). \tag{11}
\]
Assuming phase voltage \(V_1\) on the reference, the input current is
\[
I_1 = \frac{V_1}{\sqrt{3} Z_{in}}. \tag{12}
\]
The reflected secondary current follows from current division
\[
I_2 = \frac{Z_m}{Z_m + Z_{22}} I_1, \tag{13}
\]
The total developed torque \((3T_d)\) is determined as the power flowing across the air gap at synchronous speed [7]
\[
3T_d = \frac{3(I_2')^2 R_2'}{s \omega_s}. \tag{14}
\]
The motor shaft torque \((T_s)\) is then found as
\[
T_s = 3T_d - T_{FW}. \tag{15}
\]
The total input power to the motor can be calculated by
\[
P_T = \sqrt{3} V_L I_1 \cos(\omega T_1). \tag{16}
\]

III. MATLAB IMPLEMENTATION

The MATLAB code of the Appendix generates the speed-shaft torque and speed-input power graphs for use in the cost amortization study. Comments within the code serve to define the required data. Generally, variable symbols match up with those in the equations of Section II. Two exceptions because of constraints placed on variable names by MATLAB are \(T_{td}\) and \(R_{cf}\), which correlate with \(3T_d\) and \(R_c(f)\), respectively, in the equations of Section II. When selecting \(f_{max}\) (maximum stator winding frequency) and \(n_e\) (number of frequencies for analysis), an integer ratio of \(f_{max}/n_e\) is suggested so that the resulting plots correspond to an integer value of stator frequency.

Fig. 2. V/Hz control.

When this guideline is used, any interpolation between plots is simplified.

\textit{ASDstudy.m} forms a loop on stator frequency incremented over the array of \(f\) values. The line voltage is calculated to hold the rated V/Hz ratio up to rated voltage and then limited to rated value for frequencies above rated frequency in accordance with Fig. 2 [4]. The program generates arrays of speed-shaft torque \((\eta_m - T_s)\) and speed-input power \((\eta_m - P_T)\) for each value of stator frequency. The complete sets of calculated values for \(T_s\) and \(P_T\) are plotted for rated frequency; however, for other than rated frequency, only the values for speeds greater than the corresponding to breakdown torque are plotted to aid in distinguishing the various curves. In addition, the load torque \((T_L)\) characteristic is superimposed on the \(n_m - T_s\) plot.

IV. EXAMPLE STUDY

A 460-V 60-Hz four-pole three-phase induction motor has the following equivalent circuit values known from rated frequency tests:
\[
R_1 = 0.4 \ \Omega \quad R_2' = 0.5 \ \Omega \quad X_1 = 1.0 \ \Omega \quad X_2' = 1.0 \ \Omega \quad R_c = 485 \ \Omega \quad X_m = 60 \ \Omega. \tag{17}
\]
Friction and windage losses for the motor are described by
\[
T_{FW} = 0.15 + 3.62 \times 10^{-6} n_{m}^{1.5} \tag{18}
\]
where \(T_{FW}\) is in N\(\cdot\)m if \(n_m\) is in units of rpm. Based on knowledge of the motor ferromagnetic material, the eddy current losses make up 60% of the total core losses.

A. Problem Defined

The motor is directly coupled to a blower characterized by
\[
T_L = 1.11 \times 10^{-3} n_{m}^{1.5} \tag{19}
\]
when \(T_L\) is in units of N\(\cdot\)m and \(n_m\) is the shaft speed in rpm.

This blower circulates air in a building. It imperative that the air circulate 24 h/d. However, the air flow for the night-time hours would be adequate if the blower speed were reduced to 1000 rpm. Otherwise, the motor operates at rated voltage and frequency. Additional pertinent information is as follows.

1) A 20-kVA inverter is the standard suitable size.
2) The inverter cost is $80/kVA.
3) The electric energy cost is $0.09 per kW\(\cdot\)h.
4) Typical inverter efficiency is 96%.

Use \textit{ASDstudy.m} to make an economic study of this application, comparing the case of operating the blower with a
constant voltage, constant frequency installation of the motor against the case where the motor is fed by a constant V/Hz inverter so that the motor operates at 460 V, 60 Hz for 12 h/d and at 1000 rpm for the other 12 h. Specifically, determine the length of time over which the additional cost of the inverter is amortized.

B. Solution

The program (ASDstudy.m) has already been edited to reflect the data for this study. Execution of the program yields Figs. 3 and 4. Enter Fig. 3 to determine information at points A and B of operation

Point A: 460 V, 60 Hz, 1740 rpm
Point B: 262.2 V, 34.2 Hz, 1000 rpm.

The frequency for point B is an interpolation between the curve for 34 and 36 Hz. The voltage is determined by

\[(f/f_R)V_{LR} = (\frac{32.4}{60})(400) = 262.2 \text{ V.}\]

Next, enter Fig. 4 to find the input power for points A and B, knowing the speed and frequency for each point

Point A: 60 Hz, 1740 rpm, \(P_T = 12.7 \text{ kW}\)
Point B: 34.2 Hz, 1000 rpm, \(P_T = 3.4 \text{ kW}\).

The annual energy costs for operation can now be determined based on 365 d/y or 8760 h/y. Without the inverter installed, the annual cost is

\[\text{Cost 1} = (12.7 \text{ kW}) \left( \frac{8760 \text{ h}}{\text{y}} \right) \left( 0.09 \frac{\$}{\text{kW} \cdot \text{h}} \right) = \$10,012.68,\]

With the inverter installed, the motor operates half of the time at point A and the other half of the time at point B. Using the 96% inverter efficiency, the annual cost is

Point A: \(\left( \frac{12.7 \text{ kW}}{0.96} \right) \left( \frac{8760 \text{ h}}{2 \text{ yr}} \right) \left( 0.09 \frac{\$}{\text{kW} \cdot \text{h}} \right) = \$5214.94\)

Point B: \(\left( \frac{3.4 \text{ kW}}{0.96} \right) \left( \frac{8760 \text{ h}}{2 \text{ yr}} \right) \left( 0.09 \frac{\$}{\text{kW} \cdot \text{h}} \right) = \$1396.13\)

\[\text{Cost 2} = \$6611.07,\]

The annual cost saving for the inverter installation is

\[\text{Cost 1} - \text{Cost 2} = \$10,012.68 - \$6611.07 = \$3401.61 \text{ per year.}\]
ASDstudy.m—Plots shaft torque (Ts) and input power (PT) vs. speed (nm) for three-phase induction motor operation under V/Hz control.

```matlab
clear; clf;

% Equivalent circuit values for rated voltage & frequency
R1 = 0.4; R2p = 0.5; X1 = 1.0; X2p = 1.0; Rc = 485; Xm = 60;
fR = 60; VLR = 460; p = 4; % Rated voltage & frequency; poles
L1l = X1/2/p/Rf; L1lp = X2p/2/p/Rf; Lm = Xm/2/p/Rf; % Inductance values
nf = 35; fmax = 70; % no. frequencies for analysis; maximum frequency
f = linspace(fmax/nf, fmax, nf); f = [f fR]; % Frequency array

% Coefficients & exponent of F&W equation for torque in N − m
a0 = 0.15; a1 = 3.62e+6; kF = 1.8;
% Coefficients & exponent of load equation for torque in N − m
b0 = 0; b1 = 1.11e+3; kL = 1.5;
a = 0.6; % Eddy current pu portion of core losses
npts = 250; % no. points for torque-speed curve

for k = 1:nf % Calculation of performance for nf frequencies

    ws = 2/p*(2*pi*f(k)); w = 2*pi*f(k); % Synch. speed & radian frequency
    VL = f(k)/fR*VLR; % Line voltage at frequency f(k)
    if f(k)/fR <= 1; VL = f(k)/fR*VLR; else; VL = VLR; end
    wm = linspace(0, ws - 0.0001, npts); % Speed points for analysis
    for m = 1:npts % Calculation of Ts − nm for frequency f(k)
        s = (ws - wm(m))/ws;
        Rcf = (a*fR)^2 + (1 - a)*fR^2 + (1 - a)*f(k)^2*Rc;
        Z1l = R1 + j*w*L1l; Z2l = R2p/s + j*w*L2lp; Zm = j*w*Lm/Rcf/(Rcf + j*w*Lm);
        Zin = Z1l + Z2l*Zm/(Z2l + Zm);
        I1 = VL/sqrt(3)/Zin; I2p = Zm*I1/(Zm + Z2l);
        TTd = 3*(abs(I2p))**2*R2p/ws; % Total developed torque
        Ts(m) = TTd - (a0 + a1*(wm(m)*30/pi)^kF); % Shaft torque
        if Ts(m) < 0; Ts(m) = 0; end
        PT(m) = sqrt(3)*VL*abs(I1)*cos(angle(I1)); % Input power
        if Ts(m)*wm(m)/PT(m) > 100; Ts(m) = 0; end
    end
    % Plot complete curves for rated frequency, but only portions
    % break down torque speed curve
    if f(k) == fR
        figure(1); plot(wm*30/pi, Ts); grid on; hold on
        figure(2); plot(wm*30/pi, PT/1000); grid on; hold on
        figure(3); plot(wm*30/pi, eff); grid on; hold on
        figure(4); plot(wm*30/pi, rotloss); grid on; hold on
        fdev = f(2) - f(1);
        text(0.05, 0.95, ['Frequency increment: ', num2str(fdev), ' Hz', 'sc']);
        text(0.05, 0.90, ['Base curve(solidline): ', num2str(fR), ' Hz', 'sc']);
        TL = b0 + b1*(30/pi*wm) .^kL; nm = wm*max(f)/f(k)*30/pi;
        plot(nm, TL, '-. '); % Superimpose load torque plot
        else
            smax = R2p/sqrt(R1^2 + w^2*(L1l + L1lp)^2); wmmmax = (1 - smax)*ws;
            for n = 1:npts; if wm(m) >= wmmmax; break; end; end
            n1 = fix(1.1*n); n2 = fix(0.9*n); if n2 < 1; n2 = 1; end
            figure(1); plot(wm(n:npts)*30/pi, Ts(n:npts), '-'); hold on
            figure(2); plot(wm(n:npts)*30/pi, PT(n:npts)/1000, '-'); hold on
            figure(3); plot(wm(n:npts)*30/pi, eff(n1:npts), '-'); hold on
            figure(4); plot(wm(n1:npts)*30/pi, rotloss(n2:npts), '-'); hold on
            end
            end
        end
    end
end; hold off;
```

Fig. 7. MATLAB Code for ASD study.
The amortization time follows as

\[
\text{amortization time} = \frac{\text{inverter cost}}{\text{savings}} = \frac{(20 \text{ kVA})(80 \text{ kVA})}{3401.61 \frac{\$}{y}} = 0.47 \text{ y}
\]

or approximately 5.6 mo. After 5.6 mo, the inverter that cost $1600 is saving the user $3401 per year.

C. Extended Study

If the leading \%X is removed from the ten lines of code shown in the Appendix, then the program will produce two additional plots that can be used for extended understanding of ASDs. Fig. 5 is the first of the plots showing the motor efficiency with frequency as a parameter. For a particular speed and frequency of operation, this plot can be entered to determine the motor efficiency. This enlightening exercise illustrates clearly that for typical operation, the motor efficiency remains at respectable levels.

The second of the additional plots is displayed by Fig. 6, showing rotor losses with frequency as a parameter. For fan-type loads, such as used for the example of this paper, rotor heating at reduced speeds is not an issue since rotor losses are significantly reduced as speed decreases. However, in an application, such as a constant-load torque, rotor losses may remain at near the rated condition value, or possibly increase, as speed is reduced. For a self-ventilated motor, the reduced cooling air flow at low speeds can lead to rotor overheating. This set of curves provides a mechanism to quantitatively assess rotor losses at reduced speed for a qualitative judgment call on the need to add forced ventilation in certain load applications.

V. Conclusion

Because of time constraints imposed by the amount of material to be covered and possible avoidance of the cumbersome trial-and-error calculation required, an economic study to show the time for amortizing the cost of adjustable speed drives is typically not included in an electric drives course. However, economic justification is the underlying reason for many ASD applications. The presented MATLAB program provides a way to include this topic in a course with no more than half a lecture period.

The author’s experience with the amortization study has been to post the MATLAB code on a Web site after introduction in lecture to allow students ready access to an error-free copy of the program. The students are then assigned a homework problem for solution with a load profile defined like unto that of the example contained in this paper. A set of motor parameters and a load characteristic that differ from the classroom presentation are specified requiring the students to edit the code to carry out the problem solution, and thereby necessitating that the students gain some understanding of the program. The student response to this exercise is generally positive in that the work is recognized as a “real-life” engineering decision.

APPENDIX

See Fig. 7.1

REFERENCES


Jimmie J. Cathey (S’64–M’66–SM’83) was born in Whitney, TX, on May 16, 1941. He received the B.S.E.E. degree from Texas A&M University, College Station, the M.S.E.E. degree from Bradley University, Peoria, IL, and the Ph.D. degree from Texas A&M University in 1965, 1968, and 1972, respectively.

In 1965, he joined Caterpillar Tractor Company as a Research Engineer. For the periods 1968–1969 and 1972–1980, he was with Marathon LeTourneau Company, progressively holding the positions of Project Engineer, Chief Electrical Engineer, and Director—Applied Research. Since 1980, he has been with the University of Kentucky, Lexington, where he is presently a Professor of electrical engineering.

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1This file can also be downloaded from www.engr.uky.edu/~cathey/electric_mach061301.