Combined space–time block coding and eigen-space tracking in MIMO systems

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A combined space–time block coding (STBC) and eigen-space tracking (EST) scheme in multiple-input-multiple-output systems is proposed. It is proved that the STBC-EST is capable of shifting hardware complexity from the receiver to the transmitter without any bit error rate (BER) performance loss. A computational efficient EST algorithm is also proposed, which makes the STBC-EST affordable. Simulation results show that the STBC-EST with a modest feedback requirement results in a negligible BER performance loss compared with a dual system configuration.

Introduction: It is well known that a non-adaptive transmit diversity system, equipped with more transmit antennas than receive antennas, has an inherent bit error rate (BER) performance loss compared with a receive diversity system with reverse number of transmit and receive antennas (the dual system). To improve the BER performance, schemes combining transmit antenna selection (TAS) with space–time block/trellis coding (STBC/STTC) were proposed in [1, 2]. Those schemes reduce the BER degradation; however, there is still at least a 1.5 dB SNR gap between them and the corresponding dual system configuration. In this Letter, we propose a combined STBC and eigen-space tracking (EST) scheme along with a computational efficient EST algorithm. We prove that the STBC-EST achieves the same BER performance as the dual system does, so it closes the gap. Hence we suggest the STBC-EST as a way to shift the hardware complexity from the receiver to transmitter without a BER performance loss.

System and channel model: We consider a dual multiple-input-multiple-output (MIMO) channel pair, denoted by matrix $H$ and $H'$. $H$ is an $nT \times nT$ matrix, representing a channel with $nT$ transmit antennas and $nR$ receive antennas; $H'$ is an $nT \times nR$ matrix, representing a channel with $nR$ transmit and $nT$ receive antennas. The entries of both matrices are assumed to be IID complex Gaussian with zero mean and unit variance. Denoting the channel input as a vector $x$ ($nT \times 1$), output as $y$ ($nR \times 1$) and additive white Gaussian noise (AWGN) as $n$ ($nR \times 1$), the channel input/output equation can be written as

$$y = Hx + n$$

(1)

The transmit power $P$ can be expressed as

$$P = E[|x|^2]$$

(2)

where $E[\cdot]$ denotes the expectation and $|\cdot|$ denotes the conjugate transpose. We assume that the entries of $n$ are zero-mean IID Gaussian variables with variance $\sigma_n^2$, and also $nT > nR \geq 2$. Thus, a wireless channel configuration $H$ is preferred to $H'$ in the downlink, as it is much more costly to add more antennas at mobile sets than at base stations. Now we apply an STBC to both channels. It is shown in [3] that with an STBC a MIMO channel, e.g. $H$, is equivalent to a scalar channel with an effective SNR:

$$\gamma = \frac{P}{nT \sigma_n^2} \|H\|^2_F$$

(3)

where $\| \cdot \|_F$ denotes the Frobenius norm. Similarly, for $H'$:

$$\gamma' = \frac{P}{nR \sigma_n^2} \|H'\|^2_F$$

(4)

The BER performance of the STBC is completely determined by the probability distribution of this effective SNR, $\gamma$ or $\gamma'$. Note that $\|H\|^2_F$ and $\|H'\|^2_F$ have an identical $\chi^2$ distribution with a degree freedom of $2nTnR$. By comparing (3) with (4), it is clear that, because $nT > nR$, the BER performance of system $H$ is worse than that of system $H'$, which can be represented as an SNR penalty of $10 \log (nT/nR)$ dB. We refer to it as the transmit diversity SNR penalty. In [1] a combined TAS and STBC scheme was proposed to decrease this SNR penalty. It is shown that, for a system with $nT = 4$ and $nR = 2$, the TAS-STBC still has a 1.5 dB SNR penalty. This Letter proposes how this SNR penalty can be eliminated completely.

Combined STBC and EST: As seen from (3) and (4), the penalty associated with $H$ occurs because the total power $P$ is distributed into $nT$ transmit antennas. In contrast, for $H'$ system, $P$ is only distributed to $nR$ antennas. We propose a combined STBC and EST with a block diagram shown in Fig. 1. At the transmitter, the source symbol is encoded by an STBC, which is designed for $nT$ transmit antennas, i.e. $x$ is an $nR \times 1$ vector. $x$ is then preprocessed by an $nR \times nR$ matrix $W$ before it is transmitted through $nT$ antennas as an $nT \times 1$ vector $x$. Thus,

$$x = Wx$$

(5)

To keep the transmit power unchanged, we constrain the columns of $W$ to be orthonormal, i.e. $W^H W = I$. Accordingly,

$$P = E(x^H x) = E(W^H W x^H x) = E(x^H x)$$

(6)

At the receiver, a maximal ratio combining (MRC) is applied to the received $nR \times 1$ vector $y$ to recover the source symbol. The dashed box in Fig. 1 shows an equivalent $nR \times nR$ channel seen by the STBC, where the input/output equation is

$$y = HWx + n$$

(7)

Thus the equivalent SNR is

$$\tilde{\gamma} = \frac{P}{nR \sigma_n^2} \|HW\|^2_F$$

(8)

To achieve the best BER performance, we must maximise (8) with respect to $W$.

By applying a singular value decomposition (SVD), $H$ can be written as $H = UDV^H$,

$$H = UDV^H$$

(9)

where $D$ is an $nR \times nR$ diagonal matrix with the singular values of $H$, $\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{nR}}$ as its diagonal entries; $U = [u_1, \ldots, u_{nT}]$ and $V = [v_1, \ldots, v_{nR}]$, where $u_i$ and $v_i$ are, respectively, the left and right singular vectors of $H$ associated with $\sqrt{\lambda_i}$.

Denoting the range space of a matrix $A$ by ran($A$), we can prove the following theorem. For presentation brevity, we do not show the proof in this Letter, which can be provided on request.

Theorem 1: For an $nT \times nR$ matrix $W$ satisfying $W^H W = I$ and $V$ defined in (9), the following statements are equivalent:

(A) ran($W$) = ran($V$)

(B) $WV^H = V^H$

(C) $W = VX$, for some $nR \times nR$ unitary matrix $X$

(D) $W = \arg \max_{W} \|HW\|^2_F$, where $W$ is an $nT \times nR$ matrix satisfying $W^H W = I$.

The maximum equivalent SNR achieving $W$ in (8) is just the statement (D) in Theorem 1. Thus it is equivalent to statement (A), i.e. the columns of $W$ and $V$ span the same subspace. We refer to the task of tracking the subspace ran($V$) by $W$ as eigen-space tracking (EST). In short, maximising (8) is equivalent to a perfect EST.

If a perfect EST is achieved, using (C) of Theorem 1 in (8), we have

$$\tilde{\gamma} = \frac{P}{nR \sigma_n^2} \|HX\|^2_F = \frac{P}{nR \sigma_n^2} \|UDX\|^2_F = \frac{P}{nR \sigma_n^2} \sum_{i=1}^{nR} \lambda_i = \frac{P}{nR \sigma_n^2} \|H\|^2_F$$

(10)

A comparison of (10) and (4) shows that the distributions of $\tilde{\gamma}$ and $\gamma'$ are identical. Thus the BER performance of system $H$ with a combined STBC and EST is equal to that of the dual system $H'$ with an STBC. This shows a way to shift the hardware complexity, such as the antennas and RF chains from the receiver to the transmitter, without incurring any BER performance penalty.
Computation efficient EST algorithm: Here we propose a computation efficient EST algorithm. We assume the channel matrix $H$ is estimated at the receiver and fed back to the transmitter periodically. We denote the feedback as $H(k)$, $k = 1, 2, \ldots$. Initially, we randomly generate an $n_T \times n_R$ matrix $W(0)$ and set $W(0) = GS[H(0)]$, where $GS[\cdot]$ represents the Gram-Schmidt orthonormalisation. After each arrival of $H(k)$, we update $W(k)$ as follows:

$$W'(k) = H^H(k)H(k)W(k - 1)$$  \hspace{1cm} (11a)  

$$W(k) = GS[W'(k)]$$  \hspace{1cm} (11b)  

Between two consecutive feedbacks, $W(k)$ is kept constant. In fact, (11) is one iteration of the orthogonal iteration algorithm [4, chap. 7], a general method to calculate the eigen-structure of a matrix iteratively. From the convergence result for this iteration [4, chap. 7], we have

$$\text{dist}(\text{ran}(V(k)), \text{ran}(W(k))) = 0$$  \hspace{1cm} (12)  

where $\text{dist}(S_1, S_2)$ denotes the distance between subspace $S_1$ and $S_2$ [4, chap. 2]. Thus, according to the subspace distance definition,

$$\text{ran}(W(k)) = \text{ran}(V(k))$$  \hspace{1cm} (13)  

Note that the update procedure (11) needs to be performed only once per new arrival of channel coefficients $H(k)$, so the feedback rate is equal to the update rate. In contrast, a conventional iteration algorithm requires enough iterations for each arrival of $H(k)$. So the proposed EST algorithm is much more computation efficient and makes the STBC-EST affordable.

Numerical results and conclusion: We applied the STB-EST to a time-varying MIMO channel with $n_T = 4$ and $n_R = 2$ and simulated this system. The Alamouti scheme is used as the STBC and BPSK as the modulation format. The channel estimation is assumed to be perfect and fed back to the transmitter without error or delay. In theory, a perfect EST requires feedback at the symbol rate $f_s$, since the channel is continuously changing. We can ease the situation by lowering the feedback rate $f_d$ to the order of the maximum Doppler frequency shift $f_d$ and hence $f_d$ becomes much lower than and independent of $f_s$. Between two consecutive feedbacks, $W$ is kept constant and leads to mismatch between $W$ and $H$ (imperfect tracking). Since the channel is changing at most as rapidly as $f_d$, we can expect that the mismatch between $W$ and $H$ is not significant, so the performance loss due to this mismatch is not large. We investigated various values of $f_d$ relative to $f_s$. The results are shown in Fig. 2. Note that the perfect tracking curve is identical to the curve for the dual system with $n_T = 2$ and $n_R = 4$ Alamouti scheme, so the latter is not shown in this Figure.

It can be seen from Fig. 2 that the performance loss due to imperfect tracking gets larger as the SNR goes higher or equivalently as the BER goes lower. However, when $f_d = 16f_s$, the BER performance degradation relative to the perfect tracking is less than 0.2 dB in SNR evaluated at the BER range down to $10^{-7}$, which is completely satisfactory in most practical systems. Note that the feedback rate requirement is independent of the data rate. For a high rate system, where $f_d \gg f_s$, the feedback rate is not significant. In short, by employing a combined STBC and EST, we shift the hardware complexity from the receiver to transmitter with negligible BER performance loss at the cost of a computation efficient EST and a modest feedback requirement.

References