Carrier Frequency Acquisition and Tracking for OFDM Systems

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Abstract—In this paper, we present and analyze a technique for fast acquisition and accurate tracking of the carrier frequency in orthogonal frequency division multiplexing (OFDM) receivers. The scheme is based on a data-aided frequency estimation algorithm recently proposed in the literature by the authors. The presence of known symbol sequences periodically inserted in the OFDM frame allows the data demodulator to rapidly lock onto the carrier frequency during the acquisition phase, even in the presence of frequency offsets up to a few tenths of the overall signaling rate. Once acquisition is over, the circuit switches to a decision-directed mode to perform fine frequency tracking for reliable data demodulation. The algorithm performance is analyzed in terms of width of the lock-in frequency range and of lock-in probability in the acquisition mode, and of mean-square frequency estimation error in the tracking mode. Since OFDM is known to be extremely sensitive to carrier frequency errors, the impact of the carrier frequency synchronizer on the receiver error rate is also investigated.

I. INTRODUCTION

THE multicarrier transmission technique known as orthogonal frequency division multiplexing (OFDM) has recently received considerable attention for its robustness to multipath selective fading, provided by carrier-by-carrier equalization in the frequency domain [1], [2]. This technique is recommended in Europe by the ITU-R as a transmission standard for digital audio broadcasting (DAB) from satellite or terrestrial fixed stations to mobile users [3]; it was extensively studied and tested in the framework of the RACE dTTb project [4] and it is also being adopted by EBU for terrestrial digital video broadcasting (DVB-T) [5].

As already mentioned, one of the key features of OFDM lies in its increased robustness to frequency-selective multipath fading, which is characteristic of most wideband radio communications. When the number $N$ of OFDM subcarriers is large, in fact, it can easily be argued that each frequency bin in the signal spectrum spanned by a single modulated subcarrier is affected by frequency-flat, rather than frequency-selective, fading. Assuming, as is always the case in the practice, that the (time-variant) channel response does not vary significantly within the signaling symbol interval on each subcarrier, channel equalization can be easily accomplished in the frequency domain on a bin-by-bin basis, through simple amplitude and phase compensation of each (sub)carrier, according to the relevant estimated channel frequency response [6].

Conversely, as occurs for any multicarrier transmission scheme, an OFDM signal suffers from nonlinear distortion [6], and, above all, is extremely sensitive to possible uncompensated frequency offsets between the received carrier and the local oscillator, due to Doppler shifts or to the inherent instabilities of the transmit and receive references. The sensitivity to frequency errors lies in the very structure of the OFDM signal: to avoid severe system performance degradation, it is required in fact that the uncompensated frequency offset does not exceed a small fraction of the subcarrier signaling rate, which is $N$ times smaller than the overall signaling rate. This calls for an accuracy in frequency recovery hundreds or thousands times greater than that pertaining to a single carrier system with the same throughput (overall signaling rate).

Therefore, all standardization proposals for OFDM systems with a large number of subcarriers envisage the periodic insertion in the transmitted signal of a sequence of known symbols (synchronization sequence) which, in addition to other functions, has the task to facilitate acquisition and tracking of a frequency offset. The literature on carrier frequency recovery for OFDM signals is quite scarce [7]–[10], especially in the context of digital TV transmission with a large number of subcarriers. We mention here the methods described in [7], that represent modifications of a maximum likelihood (ML) closed-loop frequency tracking algorithm based on the iterative maximization of the likelihood function through the stochastic-gradient method. The ML criterion is also employed in [8] to estimate the frequency offset from observation of two consecutive synchronization sequences. Such a technique is based on the measurement of signal phase rotations occurring at corresponding symbols in the two sequences and therefore demands stationarity of the channel response at least within a synchronization sequence repetition period.

In the following, we present and analyze a different approach to frequency recovery for OFDM demodulators. Specifically, we describe a two-step frequency acquisition and tracking strategy based on an ML estimation algorithm proposed by the authors in a different context [10]. Such a method exploits, for coarse frequency acquisition, the known symbols of a single synchronization sequence (data-aided acquisition), and requires channel stationarity for a corresponding time interval. The same kind of algorithm, with a different setting of parameters allowing for a greater accuracy, is also utilized for fine frequency tracking using the decisions taken on data symbols (decision-directed tracking). The performance of the

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acquisition algorithm is assessed in terms of the extension of the lock-in frequency range and of the probability of a correct lock-in, while the tracking performance is evaluated in terms of bias and variance of the steady-state frequency error.

Finally, we provide the basic guidelines for frequency control loop design, specifying the criteria leading to the choice of the loop parameters both in the acquisition and tracking modes. In this context, we also assess, by means of a semianalytic approach, the bit-error rate (BER) sensitivity of the OFDM differential receiver to carrier frequency jitter. It is worth observing that, although in the present study we assume a Gaussian nondispersive channel, the proposed algorithm can be easily adapted to a time-varying, dispersive channel, provided that its impulse response is known or estimated by the receiver (an example of this approach can be found in [10]).

The paper is organized as follows: in the next section we review the main features of the OFDM signaling standard, and we specify the channel model to be assumed in the sequel. The degradation induced on the receiver BER by a frequency offset is evaluated in Section III, while in Section IV we outline the frequency acquisition/tracking algorithm whose application to OFDM is next illustrated in Section V. Section VI summarizes the design criteria for the frequency recovery loop, and some conclusions are finally drawn in Section VII.

II. SYSTEM OUTLINE

Fig. 1(a) depicts the baseband functional block diagram of the OFDM modulator. The transmitted signal is generated starting from blocks of $N$ differentially-encoded complex-valued digital source symbols $a_n$, $n = 0, 1, \ldots, N - 1$ (we will call them frequency-domain symbols). Fig. 1(b) also shows the sketch of the OFDM frame format akin to that of the European 802.11b-RACE project [4]. As is apparent, the frame consists of $J \gg 1$ consecutive $T$-long blocks $S_i$, $i = 0, 1, \ldots, J - 1$ of frequency-domain symbols for an overall duration $T_f = JT$.

Each block of frequency-domain symbols is used to produce a corresponding block of channel symbols $b_k$, $k = 0, 1, \ldots, N - 1$ (addressed as time-domain symbols) through an inverse discrete Fourier transform (IDFT) as follows:

$$
\delta_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_n \exp \left( \frac{j2\pi nk}{N} \right) \quad k = 0, 1, \ldots, N - 1. \tag{1}
$$

We observe that the time-domain symbols $b_k$ are output by the IDFT computer at the same rate as the frequency-domain symbols $a_n$ are input, namely, $1/T_s = N/T$, i.e., $N$ times faster than the block rate. The sequence $b_k$ is then passed through the transmission filter, whose impulse response $g_T(t)$ depends on the characteristics of the channel, particularly on channel spacing. In the following, we will assume a Nyquist-root raised-cosine spectral shape with rolloff factor $\alpha$, but actually such a shape is immaterial to our analysis. By a proper choice of the time origin, the complex envelope of the signal transmitted in the generic block can be written as

$$
x(t) = \sum_{k=0}^{N-1} b_k g_T(t - kT_s). \tag{2}
$$

At the beginning of the frame, an entire block of null frequency-domain symbols is transmitted to ease frame synchronization, while in all of the remaining blocks, the leading and trailing $N_g$ frequency-domain symbols are set at zero [$a_n = 0$, for $n \in (0, N_g - 1)$ and $n \in (N - N_g, N - 1)$] to reduce the overall bandwidth occupancy of the modulated signal (virtual carriers) [4].

Other blocks are periodically allocated to the transmission of a fixed sequence of known symbols (referred to as synchronization sequence), whose task is to provide coarse frequency synchronization as well as to permit local estimation of the channel frequency response required for channel equalization. The frequency recovery algorithm proposed in this paper is barely sensitive to the actual pattern of the synchronization sequence, the a priori knowledge of the symbols involved being sufficient for its correct operation. In the following, it will be assumed that symbols $a_n$, $n \in (N_g, N - N_g)$, pertaining to a synchronization block, belong to a quaternary phase shift keying (QPSK) constellation:

$$
a_n = \exp \{j\phi_n\}, \quad \Delta\phi_n = \text{the nth phase increment}, \quad \Delta\phi_n \in \{m\pi/2\}_{m=0}^{2^{N_g - 1}}.
$$

For simplicity and without impairing the validity of results, in the sequel we will assume that both synchronization and information-bearing sequences are made of independent, identically distributed QPSK symbols. The frequency recovery algorithm discussed in the following can be easily tailored, with straightforward modifications, to different modulation formats, such as spectrum-efficient multilevel QAM.

Fig. 2 illustrates the functional block diagram of the receiving section of the OFDM system. Under the assumption of ideal frame and carrier frequency recovery, the received signal after baseband conversion (not shown) is

$$
r(t) = x(t) \exp (j\theta) + w(t) \tag{3}
$$
where \( w(t) \) is additive white Gaussian noise (AWGN) with two-sided normalized power spectral density \( N_0/2P_s \) (\( P_s \) denoting the average signal power) and \( \theta \) is the (uncompensated) carrier phase offset.

After matched filtering, we assume that the signal is sampled at the zero-\( \text{ISI} \) instants \( t_k = kT_s \); the sequence \( p_k, k = 0, 1, \ldots, N-1 \) is then fed to a block computing an \( N \)-point discrete Fourier transform (DFT), yielding the sequence

\[
m_k = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} p_k \exp \left( \frac{-j2\pi nk}{N} \right)
\]

This sequence is applied in turn to a differential detector, whose output finally drives the decision device. We observe that, under the above assumptions

\[
p_k = \exp(j\theta)b_k + n_k
\]

where \( n_k \) represents Gaussian noise with zero-mean independent real and imaginary components, each with variance \( N_0/(2P_sT_s) \). Recalling the properties of the IDFT-DFT transform pair, we have

\[
z_m = \exp(j\theta)a_m + \mu_m, \quad m = 0, 1, \ldots, N-1
\]

where \( \mu_m \) has the same statistical characteristics as \( n_k \); the possibility of recovering the source symbols \( a_m \) through differential detection is thus easily understood. It is also apparent that the carrier phase offset \( \theta \) has no influence on the BER performance of the receiver, due to differential detection.

### III. IMPACT OF A FREQUENCY OFFSET ON RECEIVER PERFORMANCE

The issue of performance degradation of the OFDM receiver in the presence of a carrier frequency offset has been previously tackled either by simulation [6] or by approximate analysis [11]. We will pursue here a semianalytic approach to attain more accurate results.

In the presence of a fixed offset \( \Delta f \) between the carrier frequency and the local oscillator frequency, (3) modifies to

\[
r(t) = x(t) \exp[j(2\pi \Delta ft + \theta)] + w(t).
\]

Under the assumption that \( \Delta f \) is a small fraction of the baud rate \( 1/T_s \) (as is always the case in the practice), the filter in Fig. 2 can still be considered matched to the signaling pulse, and the sampler output can be written as

\[
p_k = b_k \exp[j(2\pi \Delta ft_k + \theta)] + n_k, k = 0, 1, \ldots, N-1.
\]

Recalling (4), the fast Fourier transform (FFT) block outputs the sequence

\[
z_m = \frac{e^{j\theta}}{N} \sum_{k=0}^{N-1} a_k \exp \left( -j2\pi nk/N \right)
\]

\[
1 - \exp \left\{ j2\pi N \left[ \frac{k}{NT_s} - \left( \frac{m}{NT_s} - \Delta f \right) T_s \right] \right\} + \mu_m
\]

\[
= \frac{e^{j[\theta+\pi(N-1)\Delta f]}}{N \sin(\pi\Delta f T_s)} \alpha_m
\]

\[
1 - \exp \left\{ j2\pi N \left[ \frac{k}{NT_s} - \left( \frac{m}{NT_s} - \Delta f \right) T_s \right] \right\}
\]

\[
+ \mu_m + \frac{e^{j\theta}}{N} \sum_{k \neq m} a_k \exp \left( -j2\pi nk/N \right)
\]

\[
1 - \exp \left\{ j2\pi N \left[ \frac{k}{NT_s} - \left( \frac{m}{NT_s} - \Delta f \right) T_s \right] \right\}
\]

\[
+ \mu_m + \frac{e^{j\theta}}{N} \sum_{k \neq m} a_k \exp \left( -j2\pi nk/N \right)
\]

\[
m = 0, 1, \ldots, N-1.
\]

Apparently, two different detrimental effects arise because of the frequency offset: i) the amplitude reduction of symbol \( \alpha_m \) by the factor \( \sin(\pi N\Delta f T_s)/[\sin(\pi\Delta f T_s)] \) and ii) a sort of intersymbol interference term (the addenda in the sum for \( k \neq m \)) due to the loss of orthogonality between the subcarriers. The decision variable at the differential detector output is formed as follows:

\[
\eta_m = z_m z_{m-1}^*.
\]

Since the complexity of (10) baffles any reasonable effort toward a theoretical calculation of the BER, we resort to a computer-aided semianalytic approach to evaluate the effect of a nonzero detuning \( \Delta f \), as is detailed in the following. For a given \( \Delta f \), we generate by simulation a noiseless sequence \( z_m \) as in (8) and (9), and for each pair of consecutive samples \( z_m, z_{m-1} \) we evaluate theoretically the conditional BER \( P_b(e|z_m, z_{m-1}, \Delta f) \) as a function of the energy-per-bit-to-noise-spectral-density ratio \( E_b/N_0 = P_sT_s/2N_0 \). The expression of \( P_b(e|z_m, z_{m-1}, \Delta f) \) can be derived through the procedure outlined in [12], which is similar to the approach described in [13] to calculate the error probability of a differential detector for PSK signaling. Skipping all details, we end up with

\[
P_b(e|z_m, z_{m-1}, \Delta f) = \frac{Q(a, b) + Q(c, b, a)}{4} + \frac{Q(a', b') + Q(c', b', a')}{4}
\]
where $Q(a, b)$ is the Marcum Q-function and $Q_v(b, a) \triangleq 1 - Q(b, a)$. In (11) we let

\begin{align}
    a & \triangleq \sqrt{\frac{E_b}{2N_0}} (\nu_1 - \nu_2) \\
    a' & \triangleq \sqrt{\frac{E_b}{2N_0}} (\nu'_1 - \nu'_2) \\
    b & \triangleq \sqrt{\frac{E_b}{2N_0}} (\nu_1 + \nu_2) \\
    b' & \triangleq \sqrt{\frac{E_b}{2N_0}} (\nu'_1 + \nu'_2).
\end{align}

(12a-12b)

\(\nu_1 \Delta [\overline{z}_m]^2 + [\overline{z}_{m-1}]^2\)

\(\nu'_1 = \nu_1,\)

\(\nu_2 \Delta 2R\left\{\overline{z}_m \overline{z}_{m-1} \exp\left[j\left(\Delta \phi_m - \pi / 4\right)\right]\right\}\)

\(\nu'_2 \Delta 2R\left\{\overline{z}_m \overline{z}_{m-1} \exp\left[j\left(\Delta \phi_m + \pi / 4\right)\right]\right\}

(13a-13b)

where $\Delta \phi_m$ is the $m$th transmitted information-bearing phase shift associated to the samples $\overline{z}_m$, $\overline{z}_{m-1}$. As a next step, $P_b(e|\Delta f)$ is averaged by simulation over the data pattern in $\overline{z}_m$ and $\overline{z}_{m-1}$, so as to obtain the BER $P_b(e|\Delta f)$, conditioned by $\Delta f$ only, as a function of $E_b/N_0$. Fig. 3 shows the curves of $P_b(e|\Delta f)$ versus $E_b/N_0$ for different values of the normalized frequency offset $\Delta fT_s$. The curve of the (average) BER in the presence of steady-state frequency jitter was determined assuming a Gaussian probability density function (pdf) for $\Delta f$, with zero mean and variance $\sigma^2_f$, as is approximately the case when the carrier frequency is recovered through a narrowband control loop [14]

\[ P_b(e) = \frac{1}{\sqrt{2\pi}\sigma_f} \int_{-\infty}^{\infty} P_b(e|\Delta f) \exp\left(-\frac{\Delta f^2}{2\sigma^2_f}\right) d\Delta f. \]

(14)

The integration in (14) is easily carried out with the aid of a numerical Gauss quadrature rule, leading to the results of Fig. 4, that shows a few diagrams of the BER (14) as a function of $E_b/N_0$, plotted for some values of the root mean square (RMS) normalized frequency detuning $\sigma_fT_s$. The extreme sensitivity of OFDM towards carrier frequency jitter is well documented by these curves. At a BER of $10^{-5}$, for instance, $\sigma_f/N_0$ degradation of 1 dB calls for an RMS frequency jitter of $4 \cdot 10^{-5}$ times the signaling rate. As expected, this figure is roughly $N$ times smaller than the corresponding one for differential detection of mono-carrier modulations.

**IV. REVIEW OF THE FREQUENCY ESTIMATION ALGORITHM**

We briefly outline, hereafter, the frequency estimation algorithm [10] we are going to apply to the OFDM signal.

\[ Q(a, b) \triangleq \int_0^\infty I_0(xy) \exp\left[-(y^2 + a^2)/2\right] dy, \]

where $I_0(\cdot)$ is the modified Bessel function of the first kind and order 0.

\[ \begin{align*}
    1 & \quad \text{for the BER of the "raw" uncoded link is specified for a good-quality HDTV transmission [12]}
\end{align*} \]

Consider the general problem of finding the maximum likelihood (ML) estimator of the rotation frequency $\Delta f$ of the complex-valued phasor $\exp(j2\pi\Delta f t)$, from observation of the samples

\[ \rho_i = \exp\left\{2\pi\Delta f T_s + \varphi\right\} + n_i, \quad 1 \leq i \leq Q \]

(15)

where $T_s \leq 1/(2\Delta f)$ is the sampling interval, $\varphi$ is an unknown random phase with uniform probability density in $[-\pi, \pi]$ and where $n_i$ is complex Gaussian noise as in (5). Skipping all details, we observe that the determination of the exact point where the ML function has absolute maximum turns out to be a very demanding task that cannot be pursued in a practical implementation. A search among suboptimal algorithms led to the following estimator of $\Delta f$ [10]

\[ \Delta f \approx \frac{1}{\pi T_s(M + 1)} \arg \left\{ \sum_{k=1}^M R(k) \right\} \]

(16)

where $R(k)$ denotes the estimated autocorrelation of the sequence $\rho_i$.

\[ R(k) \triangleq \frac{1}{Q - k} \sum_{i=k+1}^Q \rho_i \rho_{i-k}^* \]

(17)

and the integer $M$ is a design parameter. The value of $M$ must be chosen so as to obtain a good compromise between estimation accuracy (which is maximum when $M = Q/2$) and extension of the lock-in range of the algorithm (which increases with decreasing $M$). In the latter respect, we note that $\Delta f$ is correctly determined as long as the unwrapped
phase of the summation at the right-hand side of (16) does not exceed \( \pm \pi \), so that no ambiguity problems of the \text{arg} function arise. This limits the operating range of (16) to the interval

\[ |\Delta f| < \frac{1}{(M + 1)T_s}. \]

A thorough analysis of estimator (16) can be found in [10]. In particular, the estimator proves to be asymptotically (i.e., for signal-to-noise ratio (SNR) approaching infinity) unbiased, and its variance is found to lie remarkably close to the Cramér–Rao lower bound [10], [16], especially for values of \( M \) in the vicinity of \( Q/2 \). Estimator (16) and (17) can be applied to an OFDM signal as is detailed in the following section, provided that a suited method of modulation removal is applied [see (21) and (22)].

V. FREQUENCY RECOVERY STRATEGY FOR OFDM

A. A Two-Step Approach for Frequency Acquisition and Tracking

The high sensitivity of OFDM to frequency jitter pointed out in Section III may call for a large value of \( Q \) (and, consequently, for a large value of \( M \)) in the application of (16) and (17) to reduce to a negligible level the impact of frequency estimation errors on the receiver performance. This may entail an extremely narrow acquisition (or lock-in) range (18) which, on the contrary, should be large enough to accommodate the maximum \textit{a priori} uncertainty on the carrier frequency due to the instability of oscillators and/or to a Doppler shift. To ensure correct operation of the frequency recovery algorithm both in the acquisition and in the tracking mode, a two-step approach can be pursued. Specifically, parameters \( Q \) and \( M \) are initially set at the values \( Q_{A} \) and \( M_{A} \), so as to provide an adequate lock-in range according to (18). Once a frequency estimate has been effected with those parameter values, coarse frequency correction takes place, and the algorithm switches to tracking mode, whereby the estimator parameters are changed to \( Q_{T} \) and \( M_{T} \) (with \( M_{T} > M_{A} \)), so as to bring the steady-state RMS frequency jitter down to a level ensuring a specified BER performance degradation. Unfortunately, due to the random nature of the estimation errors, it may happen that the residual frequency offset after initial coarse estimation/correction does not fall within the operating (or hold-in) range \( |\Delta f| < [\{M_{T} + 1\}T_s]^{-1} \) of the tracking algorithm (missed lock event). A key design issue is therefore the identification of appropriate values for \( Q_{A}, M_{A}, Q_{T}, \) and \( M_{T} \) in order that the probability \( P_{ML} \) of the missed lock event be smaller than a prescribed value, as will be discussed in Section VI.

B. Frequency Tracking Loop

Fig. 5 shows the baseband equivalent of the OFDM receiver equipped with an hybrid frequency tracking loop, wherein the frequency error detector (FED) is digital, and the frequency correction is analog via a numerically controlled oscillator (NCO). The NCO frequency is updated at the end of each block period (hence at the rate \( 1/T \)) as follows. Letting \( \delta f_i \) denote the oscillation frequency of the NCO during the ith block, the signal at the matched filter input is thus affected, in that block, by the residual frequency error

\[ \delta f_i = \Delta f - \tilde{\Delta}f_i. \]

This error is estimated by the FED on the basis of algorithm (16) applied to the whole block period. Letting \( \tilde{\delta}f_i \) denote the estimate of \( \delta f_i \), the frequency of the NCO is updated for the subsequent block according to the following rule

\[ \tilde{\Delta}f_{i+1} = \tilde{\Delta}f_i + \tilde{\delta}f_i \]

so that the sequence \( \tilde{\Delta}f_i \) tends eventually to settle at \( \Delta f \).

Next, we concentrate on the FED block, which operates on the signal received within a data block. In the following, ideal frame and clock timing will be assumed. From (8) and from inspection of Fig. 5, the sequence feeding the FED in the ith block is

\[ p_k = b_k \exp [j(2\pi f_i kT_s + \theta)] + \zeta_k, \]

where \( \delta f_i \) is the ith residual frequency error to be estimated, \( \zeta_k \) represents noise with the same statistical properties of \( n_k \), and the phase \( \theta \) is immaterial to the algorithm operation. Prior to application of (16) to signal (21), the modulation affecting the latter must be removed. This entails that the channels symbols \( b_k \) be available at the receiver, as is reasonable to assume when the SNR is high enough to allow for reliable data detection, and \( \delta f_i \) has already been reduced to a small value. In this case, in fact, an estimated version \( \hat{b}_k \approx b_k \) of the sequence of channel symbols can be regenerated in the receiver through an IDFT on the estimated source data
\( \hat{a}_k \) (decision-directed tracking). We note that the symbols \( \hat{b}_k \) (as well as the \( b_k \)'s) no longer belong to a simple and geometrically regular constellation, as happens with the \( \hat{b}_k \)'s, but are scattered on the complex plane [see (1)]. In particular, the amplitude \(|b_k|\) is subject to large fluctuations as the index \( k \) varies, so that a modulation removal scheme has to take into account both phase and amplitude fluctuations in (21). This prevents from resorting to the simple modulation-canceling strategy outlined in [10] for PSK signals. After a trial-and-error process, we identified as the best modulation removal scheme the amplitude normalization and phase correction procedure that follows:

\[
\xi_k = \exp \left\{ j[\arg(p_k) - \arg(b_k)] \right\} = \frac{p_k}{|p_k|} & \frac{b_k^*}{|b_k|} \tag{22}
\]

The sequence \( \xi_k \) is input to estimator (16) in the place of \( \rho_k \), so as to obtain the \( i \)th frequency error estimate \( \hat{\delta}_f \). The latter is used in (20) to yield the estimated offset \( \hat{\Delta}_f \) that will be used for frequency compensation in the subsequent block. Note that (22) is independent of the particular signal constellation employed in the modulator, since the only information that is needed to perform modulation removal on \( p_k \) is the value of the generic time-domain symbol \( b_k \).

C. Frequency Acquisition

As already mentioned, frequency acquisition, or re-acquisition after a loss of lock, is made possible by the insertion of specialized synchronization blocks in the OFDM frame (Section II). During frequency acquisition, the tracking loop is open, and the frequency error estimator is fed with a signal affected by the entire offset \( \Delta f \). The parameters of the estimator (16), specifically the value of \( M \), are adjusted so as to enhance the acquisition capability in terms of an extended lock-in range as suggested in Section IV. A detailed example of such a design is the subject of the next section.

VI. DESIGN CRITERIA

In this section, we outline the basic steps of the frequency recovery system design, both in the acquisition and in the tracking modes, following the sensitivity analysis of Section III and the two-step approach of Section IV.

A. \( E_b/N_0 \) Degradation Due to Frequency Recovery

Once a target BER is fixed, with the aid of the diagrams of Fig. 4 we determine the standard deviation \( \sigma_f \) of the frequency tracking error not to be exceeded to attain a tolerable performance degradation with respect to the ideal case \( \sigma_f = 0 \). For instance, letting \( P_b(e) = 10^{-4} \) and requiring the performance degradation due to the residual frequency jitter not to exceed 0.3 dB, Fig. 4 yields: \( \sigma_f T_s = 3 \cdot 10^{-5} \) at \( E_b/N_0 \approx 11.3 \) dB.

B. Tracking Loop Design

For transmission over a stationary channel, the parameter \( Q_T \) should in principle be chosen as large as possible, up to the length \( N \) of the block, since the algorithm performance improves for growing \( Q_T \). In practice, an upper limit to \( Q_T \) is always dictated by the implementation complexity of the algorithm, that grows linearly with \( Q_T \) if \( M_T \) is fixed [10]. As for the value of \( M_T \), it must be selected so as to attain a good compromise between i) the variance of the residual frequency error, that must not exceed the value \( (\sigma_f T_s)^2 \) defined at step A and ii) the extension of the hold-in range, given by (18) as \( f_H = (M_T + 1)T_s^{-1} \), that must be adequately larger than \( \sigma_f T_s \) to ensure persistence of the system in the stable tracking state. Fig. 6 shows plots of the normalized error variance as a function of the frequency offset \( \delta f \). With reference to the previous example, and letting \( Q_T = 1024 \), we find that the above condition on the tracking error variance is met for values of \( M_T \) in excess of 20. Prudentially selecting \( M_T = 100 \), it is seen that the RMS error lies below \( 3 \cdot 10^{-5}T_s \) for frequency offsets in the whole hold-in range \( f_H = 9 \cdot 10^{-3}/T_s \), roughly 300 times wider than \( \sigma_f \).

C. Design for Frequency Acquisition

In the acquisition mode, the parameters \( Q_A \) and \( M_A \) must be chosen so as i) the lock-in range \( f_L = (M_A + 1)T_s^{-1} \) be wide enough to accommodate the maximum (open-loop) input frequency offset \( \Delta f \) and ii) the variance of the estimation error be sufficiently small, so as to ensure that the residual offset \( \delta f_0 = \Delta f - \hat{\Delta}f_0 \) (that represents the initial offset to be estimated by the tracking algorithm) lies within the hold-in range of the tracking loop with near-unit probability. Proceeding with the above example and keeping \( Q_A = 1024 \),

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Fig. 5. Functional block diagram of the OFDM demodulator with frequency recovery loop.
D. Evaluation of $P_{ml}$

As outlined in Section IV, the performance of the acquisition algorithm is related to $P_{ml}$, i.e., the probability that the error affecting the initial estimate of $\Delta f$ exceeds the limits $\pm f_H$ of the tracking hold-in range. The evaluation of $P_{ml}$ can be carried out as follows. Let $\Psi$ denote the argument of the complex number $R_{M_A}$ representing the sum of the $M_A$ lags of the autocorrelation $R(k)$ of the received signal as in (16), namely, $R_{M_A} = \sum_{k=1}^{M_A} R(k)$. In principle, for a given value of $\Delta f$, $P_{ml}$ can be evaluated as the probability that $\Psi$ lies outside the hold-in interval of the tracker, i.e., $\Psi \notin (\pi f_L + 2\pi f_L/\pi f_H \pm f_L)$. Unfortunately, such a problem is analytically untractable due to the complexity of (16); however, we observed from simulations that at medium-to-high $E_b/N_0$ ratios, $R_{M_A}$ bears Gaussian statistics to a good approximation [12]. Under this assumption, we are led to the classical problem of evaluating the probability that a Gaussian vector falls outside an angular sector [13]. This evaluation is in general involved, but some measures can be taken to arrive at a reasonably simple computation procedure. We skip all further details concerning the evaluation of $P_{ml}$; for the interested reader, the main steps leading to the results presented in the following are summarized in the Appendix.

Since with $Q_A = 1024$ and $M_A = 10$, as in the example above, the probability $P_{ml}$ takes on extremely small values that cannot be determined by straight Monte Carlo simulation, we verified our Gaussian approximation in a few “artificial test cases.” As an example, Fig. 7 shows the diagram of $P_{ml}$ versus the normalized initial frequency offset $\Delta f T_s$ for $Q_A = 16$, $M_A = 4$ and $Q_T = 1024$, $M_T = 10$. The lower solid-line curve represents theoretical results, while the much more demanding Monte Carlo simulations results are indicated by small squares (both diagrams are obtained at $E_b/N_0 = 7$ dB). More significant results (obtained by theory only) are represented by the upper solid curve, pertaining to $E_b/N_0 = 10$ dB: even with such an extremely short acquisition window ($Q_A = 16$), it is seen that for $|\Delta f T_s|$ up to approximately $0.2/T_s$, $P_{ml}$ does not exceed $10^{-4}$, a value that, also in view of the high repetition rate of the synchronization sequences, seems to ensure a correct (re)acquisition capability of the receiver.

VII. CONCLUSION

We have shown that application of the frequency estimation algorithm described in [10], in the form of a two-step procedure for (open-loop) acquisition and (closed-loop) tracking, can make the OFDM transmission technique resistant to those frequency offsets that may be encountered in a low-cost system implementation with typical oscillator instabilities. In particular, the proposed carrier recovery scheme allows for reliable frequency acquisition within the range $\pm 2/10$ of the overall symbol rate even with extremely short synchronization sequences. This also suggests that the net spectrum efficiency of the proposed OFDM standards could be further improved by reducing the length of such sequences down to a few tens of symbols within the frame format.

APPENDIX

Following the notation of Fig. 8, the first step in the evaluation of $P_{ml}$ is a reference frame rotation leading to the new orthogonal frame $xOy$, where the direction of the $x$ axis is determined by the vector $\bar{R}_{M_A} \triangleq \sum_{k=1}^{M_A} \exp \{j2\pi k\Delta f T_s\}$.

Fig. 6. Plots of the frequency error variance versus frequency offset.

Fig. 7. Plot of $P_{ml}$ versus $\Delta f T_s$. $Q_A = 16$, $M_A = 4$, $Q_T = 1024$, $M_T = 10$. 
Fig. 8. Quantities involved in the computation of \( P_{\text{nl}} \).

i.e., the noiseless version of the vector \( R_{MA} \) defined in the text. The vector \( R_{MA} \) is thus decomposed into the sum of a useful component \( \tilde{R}_{MA} \) and a noise component \( X' + jY \). Assuming joint Gaussian statistics for \( X' \) and \( Y \), and letting \( X = \left| \tilde{R}_{MA} \right| + X' \), it is straightforward [12] to derive the pdf \( p_{\Phi}(\phi) \) of the angle \( \Phi \) between \( R_{MA} \) and \( \tilde{R}_{MA} \), once the mean values

\[
\eta_X = E\{X\}, \quad \eta_Y = E\{Y\}
\]
and the variances

\[
\sigma_X^2 = E\{(X - \eta_X)^2\}, \quad \sigma_Y^2 = E\{(Y - \eta_Y)^2\}
\]

as well as the correlation coefficient

\[
\rho_{XY} = E\{(X - \eta_X)(Y - \eta_Y)/\sigma_X\sigma_Y\}
\]
of \( X \) and \( Y \) are known.

\[
p_{\Phi}(\phi) = \frac{1}{\sqrt{1 - \rho_{XY}^2}} \exp \left[ \frac{-1}{2(1 - \rho_{XY}^2)} \left( \frac{\eta_X}{\sigma_X^2} + \frac{\eta_Y}{\sigma_Y^2} - 2\rho_{XY} \frac{\eta_X\eta_Y}{\sigma_X\sigma_Y} \right) \right]
\]

\[
\times \sqrt{\frac{\cos^2 \phi + \sigma_Y^2}{\sigma_X^2} + \frac{\sin^2 \phi}{\sigma_Y^2} - 2\rho_{XY} \frac{\sin \phi \cos \phi}{\sigma_X\sigma_Y}} \cdot \left[ 1 + \sqrt{2\pi} D(\phi) \cdot \exp \left[ \frac{D^2(\phi)}{2} \right] \cdot \{1 - Q[D(\phi)]\} \right]
\]

where \( Q(x) \equiv (2\pi)^{-1/2} \int_{-\infty}^{x} \exp \{-y^2/2\} \, dy \), and where

\[
D(\phi) \equiv \frac{1}{\sqrt{1 - \rho_{XY}^2}} \left[ \frac{\eta_X}{\sigma_X^2} \cos \phi + \frac{\eta_Y}{\sigma_Y^2} \sin \phi - \rho_{XY} \frac{\eta_X \eta_Y}{\sigma_X \sigma_Y} \cos \phi \right]
\]

\[
\sqrt{\frac{\cos^2 \phi}{\sigma_X^2} + \frac{\sin^2 \phi}{\sigma_Y^2} - 2\rho_{XY} \frac{\sin \phi \cos \phi}{\sigma_X \sigma_Y}}
\]

Considering the definitions above, we have immediately

\[
\eta_X = \left| \tilde{R}_{MA} \right|, \quad \eta_Y = 0
\]

The most tedious part of the computation is now the evaluation of \( \sigma_X^2, \sigma_Y^2 \), and \( \rho_{XY} \). Such moments can be derived from those relevant to the "noise"

\[ E\{\cdot\} \] stands for statistical expectation.

vector \( A + jB = R_{MA} - \tilde{R}_{MA} \) as follows:

\[
\sigma_X^2 = \cos^2 \omega \sigma_A^2 + \sin^2 \omega \sigma_B^2
\]

\[
+ 2\rho_{AB} \sigma_A \sigma_B \sin \omega \cos \omega
\]

\[
\sigma_Y^2 = \cos^2 \omega \sigma_B^2 + \sin^2 \omega \sigma_A^2
\]

\[
- 2\rho_{AB} \sigma_A \sigma_B \sin \omega \cos \omega
\]

\[
\rho_{XY} = \sin \omega \cos \omega \left( \sigma_A^2 - \sigma_B^2 \right) - (\cos^2 \omega - \sin^2 \omega) \sigma_A \sigma_B \rho_{AB}
\]

\[
\sigma_X \sigma_Y
\]

where \( \omega \equiv \angle \tilde{R}_{MA} \).

Referring the reader to [12] for a detailed computation of the variances \( \sigma_X^2, \sigma_Y^2 \), and of the correlation coefficient \( \rho_{AB} \), we provide here the final results

\[
\sigma_A^2 = 2\sigma^2 \sum_{k=1}^{M} \sum_{l=1}^{M} \min\left(\frac{\left(N - l\right)(N - k)}{(N - k)(N - l)} \right)
\]

\[
\cdot \cos\left[2\pi \Delta fT_s(k - l)\right]
\]

\[
+ 2\sigma^2 \sum_{k=1}^{M} \sum_{l=1}^{M} \frac{(N - k - l)u(N - k - l)}{(N - k)(N - l)}
\]

\[
\cdot \cos\left[2\pi \Delta fT_s(k + l)\right] + 2\sigma^4 \sum_{k=1}^{M} \frac{1}{N - k}
\]

\[
\sigma_B^2 = 2\sigma^2 \sum_{k=1}^{M} \sum_{l=1}^{M} \min\left(\frac{\left(N - l\right)(N - k)}{(N - k)(N - l)} \right)
\]

\[
\cdot \cos\left[2\pi \Delta fT_s(k - l)\right]
\]

\[
- 2\sigma^2 \sum_{k=1}^{M} \sum_{l=1}^{M} \frac{(N - k - l)u(N - k - l)}{(N - k)(N - l)}
\]

\[
\cdot \cos\left[2\pi \Delta fT_s(k + l)\right] + 2\sigma^4 \sum_{k=1}^{M} \frac{1}{N - k}
\]

\[
\rho_{AB} = 2\sigma^2 \sum_{k=1}^{M} \sum_{l=1}^{M} \min\left(\frac{\left(N - l\right)(N - k)}{(N - k)(N - l)} \right)
\]

\[
\cdot \sin\left[2\pi \Delta fT_s(k - l)\right]
\]

\[
+ 2\sigma^4 \sum_{k=1}^{M} \sum_{l=1}^{M} \frac{(N - k - l)u(N - k - l)}{(N - k)(N - l)}
\]

\[
\cdot \sin\left[2\pi \Delta fT_s(k + l)\right]
\]

where \( u(\cdot) \) is the unit step sequence. The probability \( P_{\text{nl}} \) can now be evaluated as the integral of the pdf \( p_{\Phi}(\phi) \) outside the interval \((-\gamma, \gamma)\), where, recalling the discussion of Section VI-D, \( \gamma \) is defined as

\[
\gamma \equiv \pi [(M_A + 1)/(MT + 1)]
\]

\[
P_{\text{nl}} = \int_{-\gamma}^{\gamma} p_{\Phi}(\phi) \, d\phi + \int_{\gamma}^{\pi} p_{\Phi}(\phi) \, d\phi.
\]

It is worth noting that the integration interval is fixed and does not depend on the particular value of \( \Delta f \) we are considering. This invariance is a result of the preliminary rotation we have carried out to align our frame of reference with \( \tilde{R}_{MA} \). Should we have not performed such an operation, we would have been forced to consider directly the angle \( \Psi \).
formed by $R_{MA}$ with the real axis, as stated in the main text. Consequently, the pdf $p_\phi(\phi)$ should have been integrated on an interval dependent on $\Delta f$ (see Section VI-D), and a troublesome "folding" procedure should have been therefore taken up in those cases leading to an integration interval partially lying outside the 2\pi-wide definition interval of $p_\phi(\phi)$. With the above approach, on the contrary, the resulting $p_\phi(\phi)$ is a "well-behaved" function of $\phi$, and therefore a simple numerical quadrature formula is sufficient to compute the integrals in (27).

REFERENCES


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