CHAPTER SEVEN

Class A Amplifiers

7.1 INTRODUCTION

The class A amplifier is typically used as the first amplification stage of a receiver or transmitter where minimum distortion is desired. This comes with a cost of relatively low efficiency. Since the first stages in an amplifier chain handle low-power levels, the low efficiency of these amplifiers actually wastes little power. The variety of amplifier classes are described in [1] and will be covered more extensively in a later chapter. The primary properties of importance to class A amplifier design are gain, bandwidth control, stability, and noise figure. These are the topics that will be considered here.

7.2 DEFINITION OF GAIN [2]

In low-frequency circuits, gain is often thought of in terms of voltage or current gain, such as the ratio of the output voltage across the load to the input applied voltage. At radio frequencies it is difficult to directly measure a voltage, so typically some form of power gain is used. But once the notion of power is introduced, there are several definitions of power gain that might be used.

1. **Power gain.** This is the ratio of power dissipated in the load, $Z_L$, to the power delivered to the input of the amplifier. This definition is independent of the generator impedance, $Z_G$. Certain amplifiers, especially negative resistance amplifiers, are strongly dependent on $Z_G$.

2. **Available gain.** This is the ratio of the amplifier output power to the available power from the generator source. This definition depends on $Z_G$ but is independent of $Z_L$.

3. **Exchangeable gain.** This is the ratio of the output exchangeable power to the input exchangeable power. The exchangeable power of the source is
defined as
\[ P = \frac{|V|^2}{4|\Re\{Z_G\}|}, \quad \Re\{Z_G\} \neq 0 \]  
\hspace{1cm} (7.1)

For negative resistance amplifiers \( P < 0 \)! Furthermore this definition is independent of \( Z_L \).

4. **Insertion gain.** This is the ratio of output power to the power that would be dissipated in the load if the amplifier were not present. There is a problem in applying this definition to mixers or parametric upconverters where the input and output frequencies differ.

5. **Transducer power gain.** This is the ratio of the power delivered to the load to the available power from the source. This definition depends on both \( Z_G \) and \( Z_L \). It gives positive gain for negative resistance amplifiers as well. Since the characteristics of real amplifiers change when either the load or generator impedance is changed, it is desirable that the gain definition reflect this characteristic. Thus the transducer power gain definition is found to be the most useful.

### 7.3 Transducer Power Gain of a Two-Port

The linear two-port circuit in Fig. 7.1 can be analyzed with the help of Fig. 7.2 and is characterized by its impedance parameters:

\[ V_1 = z_{11}I_1 + z_{12}I_2 \]  
\hspace{1cm} (7.2)

\[ V_2 = z_{21}I_1 + z_{22}I_2 \]  
\hspace{1cm} (7.3)

\[ \begin{align*}
V_1 & = z_{11}I_1 + z_{12}I_2 \\
V_2 & = z_{21}I_1 + z_{22}I_2
\end{align*} \]

**FIGURE 7.1** Two-port circuit expressed in impedance parameters.

\[ \begin{align*}
Z_G & \quad b_G \\
\Gamma_G & \quad a_1
\end{align*} \]

**FIGURE 7.2** Equivalent circuit to determine the input available power.
But the relationship between the port-2 voltage and current is determined by the load impedance:

\[ V_2 = -I_2 Z_L \]  

(7.4)

Substitution of this for \( V_2 \) in Eq. (7.3) gives the input impedance. This is dependent on both the contents of the two-port itself and the load:

\[ Z_{\text{in}} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L} \]  

(7.5)

This will be used to determine the transducer power gain. The power delivered to the load is \( P_2 \):

\[ P_2 = \frac{1}{2} |I_2|^2 \Re \{ Z_L \} \]  

(7.6)

Since the power available from the source is

\[ P_{\text{1a}} = \frac{|V_G|^2}{8 \Re \{ Z_G \}} \]  

(7.7)

the transducer power gain is

\[ G_T = \frac{P_2}{P_{\text{1a}}} \]  

(7.8)

\[ \frac{4 \Re \{ Z_L \} \Re \{ Z_G \} |z_{21}|^2}{|Z_G + z_{11})(Z_L + z_{22}) - z_{21}z_{21}|^2} \]  

(7.9)

Similar expressions can be obtained for \( y, h, \) or \( g \) parameters by simply replacing the corresponding \( z_{ij} \) with the desired matrix elements and by replacing the \( Z_G \) and \( Z_L \) with the appropriate termination. However, for radio frequency and microwave circuits, scattering parameters are the most readily measured quantities. The transducer power gain will be found in terms of the scattering parameters in the following section.

### 7.4 POWER GAIN USING S PARAMETERS

The available power, \( P_a \), when the input of the two-port circuit is matched with \( \Gamma_i = \Gamma_{G*} \), was given by Eq. (4.124) in Chapter 4.

\[ P_a = \frac{\frac{1}{2} |b_G|^2}{1 - |\Gamma_G|^2} \]  

(7.10)

At the output side of the circuit, the power delivered to the load is given by the following:

\[ P_L = \frac{1}{2} |b_2|^2 (1 - |\Gamma_L|^2) \]  

(7.11)

The transducer gain is simply the ratio of Eq. (7.11) to Eq. (7.10):

\[ G_T = \frac{|b_2|^2}{|b_G|^2} (1 - |\Gamma_L|^2)(1 - |\Gamma_G|^2) \]  

(7.12)
As this stands, $b_2$ and $b_G$ are not very meaningful. However, this ratio can be expressed entirely in terms of the known $S$ parameters of the two-port circuit. From the description of the $S$ parameters as a matrix corresponding to forward- and backward-traveling waves, the two-port circuit can be represented in terms of a flow graph. Each branch of the flow graph is unidirectional and the combination describes the $S$ matrix completely. The presumption is that the circuit is linear. The problem of finding $b_2/b_G$ can be done using either algebra or some flow graph reduction technique. The classical method developed for linear systems is Mason’s nontouching loop rules. The method shown below is easier to remember, but it is more complicated to administer to complex circuits that require a computer analysis. For the relatively simple graph shown in Fig. 7.3, the simpler method works well. This method of flow graph reduction is based on four rules:

Rule 1. The cascade of two branches in series can be reduced to one branch with the value equal to the product of the two original branches (Fig. 7.4a).

**FIGURE 7.3** Flow graph equivalent of the two-port circuit in Fig. 7.2.

**FIGURE 7.4** Flow graph reduction rules for (a) two-series branches, (b) two-shunt branches, (c) a self-loop, and (d) splitting a node.
Rule 2. Two parallel branches can be reduced to one branch whose value is the sum of the two original branches (Fig. 7.4b).

Rule 3. As illustrated in Fig. 7.4c, a self-loop with value $Y$ with an incoming branch $X$ can be reduced to a single line of value

$$\frac{X}{1 - Y} \quad (7.13)$$

Rule 4. The transfer function remains unchanged if a node with one input branch and $N$ output branches can be split into two nodes. The input branch goes to each of the new nodes. Similarly the transfer function remains unchanged if a node with one output branch and $N$ input branches can be split into two nodes. The output branch goes to each of the new nodes (Fig. 7.4d).

These rules can be used to finish the calculation of the transducer power gain of Eq. (7.12) by finding $b_2/b_G$. The first step in this reduction is the splitting of two nodes shown in Fig. 7.5a by use of rule 4. This forms a self-loop in the right-hand side of the circuit. The lower left-hand node is also split into two nodes (Fig. 7.5b). The incoming branches to the self-loop on the right-hand side are modified by means of rule 3 (Figs. 7.5c). In the same figure another self-loop is made evident on the left-hand side. In this case there are two incoming branches modified by the self-loop. Use of rule 3 produces Fig. 7.5d. Splitting the node by means of rule 4 results in Fig. 7.5e. The resulting self-loop modifies the incoming branch on the left-hand side (rule 3). The result is three branches in series (rule 1), so the transfer function can now be written by inspection:

$$b_2 = \frac{b_G}{1 - \frac{1 - \Gamma_G S_{11}}{S_{21} S_{12} \Gamma_G \Gamma_L}} \cdot \frac{S_{21}}{1 - \Gamma_L S_{22}}$$

$$\frac{b_2}{b_G} = \frac{S_{21}}{(1 - \Gamma_L S_{22})(1 - \Gamma_G S_{11}) - S_{12} S_{21} \Gamma_G \Gamma_L} \quad (7.14)$$

This ratio can be substituted into the transducer power gain expression (7.12). Thus the transducer power gain is known in terms of scattering parameters of the two-port and the terminating reflection coefficients:

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_G|^2)(1 - |\Gamma_L|^2)}{|(1 - \Gamma_L S_{22})(1 - \Gamma_G S_{11}) - S_{12} S_{21} \Gamma_G \Gamma_L|^2} \quad (7.15)$$

This is the full equation for the transducer power gain. Other expressions making use of approximations are strictly speaking a fiction, though this fiction is sometimes used to characterize certain transistors. For example, unilateral power gain is found by setting $S_{12} = 0$. In real transistors $S_{12}$ should be small, but it is never
Maximum gain is obtained when both the input and output ports are simultaneously matched. One way to achieve this is to guess at a $\Gamma_L$ and calculate $\Gamma_i$ (Fig. 7.6). The generator impedance then is made to match the complex conjugate of $\Gamma_i$. With this new value of $\Gamma_G$, a new value of $\Gamma_O$ is found. Matching this to
\[ \Gamma_L = \text{the that } \Gamma_L \text{ changes. This iterative process continues until both sides of the circuit are simultaneously matched.} \]

A better way is to recognize this as basically a problem with two equations and two unknowns. Simultaneous match forces the following two requirements:

\[ \Gamma_i = \Gamma_G^* = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - \Gamma_LS_{22}} \quad (7.17) \]

\[ \Gamma_O = \Gamma_L^* = S_{22} + \frac{S_{21}S_{12}\Gamma_G}{1 - \Gamma_GS_{11}} \quad (7.18) \]

Since both of these equations have to be satisfied simultaneously, finding \( \Gamma_G \) and \( \Gamma_L \) requires solution of two equations with two unknowns. These can be written in terms of the determinate of the \( S \) matrix \( \Delta \) as follows:

\[ \Gamma_G^* = \frac{S_{11} - \Gamma_LS_{11}S_{22} + S_{12}S_{21}\Gamma_L}{1 - \Gamma_LS_{22}} \quad (7.19) \]

\[ \Gamma_L^* = \frac{S_{22} - \Gamma_G\Delta}{1 - \Gamma_GS_{11}} \quad (7.20) \]

Substitution of Eq. (7.20) into Eq. (7.19) eliminates \( \Gamma_L \):

\[ \Gamma_G^* = \frac{S_{11}(1 - \Gamma_G^*S_{11}) - \Delta(S_{22}^* - \Gamma_G^*\Delta^*)}{1 - \Gamma_G^*S_{11} - |S_{22}|^2 + S_{22}\Delta^*\Gamma_G^*} \quad (7.21) \]

This expression can be rearranged in the usual quadratic form. After taking the complex conjugate, this yields the following:

\[ \Gamma_G^2(S_{22}\Delta - S_{11}) + \Gamma_G(1 - |S_{22}|^2 + |S_{11}|^2 - |\Delta|^2) - S_{11}\Delta^*S_{22} = 0 \quad (7.22) \]

This equation can be rewritten in the form

\[ 0 = -\Gamma_G^2C_1 + \Gamma_GB_1 - C_1^* \quad (7.23) \]

where

\[ C_1 = S_{11} - \Delta S_{22}^* \quad (7.24) \]

\[ B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad (7.25) \]
The required generator reflection coefficient for maximum gain can be found:

\[
\Gamma_{Gm} = \frac{C^*_1}{2|C_1|^2} \left[ B_1 \pm \sqrt{B_1^2 - 4|C_1|^2} \right] \tag{7.26}
\]

In similar fashion the load reflection coefficient (impedance) for maximum gain is

\[
\Gamma_{Lm} = \frac{C^*_2}{2|C_2|^2} \left[ B_2 \pm \sqrt{B_2^2 - 4|C_2|^2} \right] \tag{7.27}
\]

where

\[
C_2 = S_{22} - \Delta S^*_{11} \tag{7.28}
\]

\[
B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \tag{7.29}
\]

The parameters \( B_i \) and \( C_i \) are determined solely from the scattering parameters of the two-port. The \(-\) sign is used when \( B_i > 0 \), and the \( +\) sign is used when \( B_i < 0 \). Once the terminating reflection coefficients are known, the corresponding impedances may be determined:

\[
Z_G = Z_0 \frac{1 + \Gamma_G}{1 - \Gamma_G} \tag{7.30}
\]

\[
Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} \tag{7.31}
\]

### 7.6 STABILITY

A stable amplifier is an amplifier where there are no unwanted oscillations anywhere. Instability outside the operating band of the amplifier can still cause unwanted noise and even device burnout. Oscillations can only occur when there is some feedback path from the output back to the input. This feedback can result from an external circuit, from external feedback parasitic circuit elements, or from an internal feedback path such as through \( C_{\mu} \) in a common emitter bipolar transistor. Of these three sources, the last is usually the most troublesome. The following sections describe a method for determining transistor stability and some procedures to stabilize an otherwise unstable transistor.

#### 7.6.1 Stability Circles

The criteria for unconditional stability require that \( |\Gamma_i| \leq 1 \) and \( |\Gamma_o| \leq 1 \) for any passive terminating loads. A useful amplifier may still be made if the terminating loads are carefully chosen to stay out of the unstable regions. It is helpful to find the borderline between the stable and the unstable regions. For the input side, this is done by finding the locus of points of \( \Gamma_L \) that will give \( |\Gamma_i| = 1 \). The
borderline between stability and instability is found from Eq. (7.17) and (7.19) when $|\Gamma_i| = 1$:

$$1 = \left| \frac{S_{11} - \Delta \Gamma_L}{1 - \Gamma_L S_{22}} \right|$$ (7.32)

This equation can be squared and then split up into its complex conjugate pairs:

$$(1 - \Gamma_L S_{22})(1 - \Delta^* \Gamma^*_L) = (S_{11} - \Delta \Gamma_L)(S^*_{11} - \Delta^* \Gamma^*_L)$$ (7.33)

The coefficients of the different forms of $\Gamma_L$ are collected together:

$$|\Gamma_L|^2(|S_{22}|^2 - |\Delta|^2) + \Gamma_L(\Delta S^*_{11} - S_{22}) + \Gamma^*_L(S_{11} \Delta^* - S^*_{22})$$

$$= |S_{11}|^2 - 1$$ (7.34)

$$|\Gamma_L|^2 + \Gamma_L \left( \frac{\Delta S^*_{11} - S_{22}}{|S_{22}|^2 - |\Delta|^2} \right) + \Gamma^*_L \left( \frac{S_{11} \Delta^* - S^*_{22}}{|S_{22}|^2 - |\Delta|^2} \right)$$

$$= \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2}$$ (7.35)

Division of Eq. (7.35) by the coefficient of $|\Gamma_L|^2$ shows that this equation can be put in a form that can be factored by completing the square. The value $|m|^2$ is added to both sides of the equation:

$$(\Gamma_L + m^*)(\Gamma^*_L + m) = |\Gamma_L|^2 + \Gamma_L m + \Gamma^*_L m^* + |m|^2 + \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2}$$ (7.36)

where

$$m \triangleq \frac{\Delta S^*_{11} - S_{22}}{|S_{22}|^2 - |\Delta|^2}$$ (7.37)

Substitution of Eq. (7.37) into Eq. (7.36) upon simplification yields the following factored form:

$$\left( \Gamma_L + \frac{\Delta^* S_{11} - S^*_{22}}{|S_{22}|^2 - |\Delta|^2} \right) \left( \Gamma^*_L + \frac{\Delta S^*_{11} - S_{22}}{|S_{22}|^2 - |\Delta|^2} \right) = \frac{|S_{12} S_{21}|^2}{(|S_{22}|^2 - |\Delta|^2)^2}$$ (7.38)

This is the equation of a circle whose center is

$$C_L = \frac{S_{11} \Delta^* - S^*_{22}}{|\Delta|^2 - |S_{22}|^2}$$ (7.39)

The radius of the load stability circle is

$$r_L = \left| \frac{S_{21} S_{12}}{|\Delta|^2 - |S_{22}|^2} \right|$$ (7.40)
The center and radius for the generator stability circle can be found in the same way by analogy:

\[
C_G = \frac{S_{22} \Delta^* - S_{11}^*}{|\Delta|^2 - |S_{11}|^2} \quad (7.41)
\]

\[
r_G = \left| \frac{S_{21} S_{12}}{|\Delta|^2 - |S_{11}|^2} \right| \quad (7.42)
\]

These two circles, one for the load and one for the generator, represent the borderline between stability and instability. These two circles can be overlayed on a Smith chart. The center of the circle is located at the vectorial position relative to the center of the Smith chart. The “dimensions” for the center and radius are normalized to the Smith chart radius (whose value is unity).

The remaining issue is which side of these circles is the stable region. Consider first the load stability circle shown in Fig. 7.7. If a matched \( Z_0 = 50 \, \Omega \)

FIGURE 7.7 Illustration of the stability circles where the shaded region is unstable.
transmission line were connected directly to the output port of the two-port circuit, then $\Gamma_L = 0$. This load would be located in the center of the Smith chart. Under this condition, Eq. (7.17) indicates that $\Gamma_i = S_{11}$. If the known value of $|S_{11}| < 1$, then $|\Gamma_i| < 1$ when the load is at the center of the Smith chart. If one point on one side of the stability circle is known to be stable, then all points on that side of the stability circle are also stable. The same rule would apply to the generator side when it is replaced by a matched load $= Z_0$.

Unconditional stability requires that both $|\Gamma_i| < 1$ and $|\Gamma_o| < 1$ for any passive load and generator impedance attached to the ports. In this case, if $|S_{11}| < 1$ and $|S_{22}| < 1$, the stability circles would lie completely outside the Smith chart. Conditional stability occurs when at least one of the stability circles intersects the Smith chart. As long as the load and source impedances are on the stable side of the stability circle, stable operation occurs. This choice may not, and usually will not, coincide with the generator and load impedance for maximum transducer power gain as given by Eqs. (7.26) and (7.27). Avoiding unstable operation will usually require compromising the maximum gain for a slightly smaller but often acceptable gain. Clearly, using an impedance too close to the edge of the stability circle can result in unstable operation because of manufacturing tolerances.

### 7.6.2 Rollett Criteria for Unconditional Stability

It is often useful to determine if a given transistor is unconditionally stable for any pair of passive impedances terminating the transistor. The two conditions necessary for this are known as the Rollett stability criteria [3] and are given as follows:

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \geq 1$$  \hspace{1cm} (7.43)

$$|\Delta| \leq 1$$  \hspace{1cm} (7.44)

Rollett’s original derivation was done using any one of the volt–ampere immittance parameters, $z$, $y$, $h$, or $g$. Subsequently his stability equations were expressed in terms of $S$ parameters as shown in Eqs. (7.43) and (7.44). Others arrived at stability conditions that appeared different from these, but it was pointed out that most of these alternate formulations were equivalent to those in Eqs. (7.43) and (7.44) [4]. The derivation of these two quantities will be given in this section.

The first of these equations is based on unconditional stability occurring when the load stability circle lies completely outside the Smith chart when $|S_{11}| < 1$, that is,

$$|C_L| - r_L \geq 1$$  \hspace{1cm} (7.45)

or

$$r_L - |C_L| \geq 1$$  \hspace{1cm} (7.46)
where Eq. (7.46) describes the case where the stability circle contains the entire Smith chart within it. Substitution of Eqs. (7.39) and (7.40) into Eq. (7.45) gives

$$\frac{|S_{22} - S_{11}^* \Delta| - |S_{12} S_{21}|}{|| \Delta \|^2 - |S_{22}|^2} \geq 1$$

(7.47)

Squaring Eq. (7.47) gives the following:

$$|S_{22} - S_{11}^* \Delta| - |S_{12} S_{21}| \geq || \Delta \|^2 - |S_{22}|^2$$

(7.48)

$$|S_{22} - S_{11}^* \Delta|^2 - 2|S_{12} S_{21}| |S_{22} - S_{11}^* \Delta|$$

$$+ |S_{12} S_{21}|^2 \geq || \Delta \|^2 - |S_{22}|^2$$

(7.49)

$$2|S_{12} S_{21}| |S_{22} - S_{11}^* \Delta| \leq -|| \Delta \|^2 - |S_{22}|^2$$

$$+ |S_{12} S_{21}|^2 + |S_{22} - S_{11}^* \Delta|^2$$

(7.50)

The last term on the right-hand side of Eq. (7.50) can be expanded:

$$|S_{22} - S_{11}^* \Delta|^2 = (S_{22} - S_{11}^* \Delta)(S_{22}^* - S_{11} \Delta^*)$$

$$= |S_{22}|^2 - S_{11} S_{22} \Delta^* - S_{11}^* S_{22}^* \Delta + |S_{11}|^2 |\Delta|^2$$

(7.51)

$$= |S_{22}|^2 + |\Delta|^2 |S_{11}|^2 - |S_{11} S_{22}|^2$$

$$+ (S_{11} S_{22} S_{12}^* S_{21} - |S_{11} S_{22}|^2 + S_{11}^* S_{22}^* S_{12} S_{21})$$

(7.52)

Now expansion of $|\Delta|^2$ gives the following:

$$|\Delta|^2 = (S_{11} S_{22} - S_{12} S_{21})(S_{11}^* S_{22}^* - S_{12}^* S_{21}^*)$$

$$= |S_{11} S_{22}|^2 + |S_{12} S_{21}|^2 - S_{11} S_{22} S_{12}^* S_{21} - S_{11}^* S_{22}^* S_{12} S_{21}$$

(7.53)

By subtracting $|S_{12} S_{21}|$ inside the parenthesis in Eq. (7.52) and adding the same value outside the parenthesis, the quantity inside the parenthesis is equivalent to $|\Delta|^2$ given in Eq. (7.53). Thus Eq. (7.52) can be factored as shown below:

$$|S_{22} - S_{11}^* \Delta|^2 = |S_{22}|^2 + |\Delta|^2 |S_{11}|^2 - |S_{11} S_{22}|^2 + |S_{12} S_{21}|^2 - |\Delta|^2$$

$$= |S_{12} S_{21}|^2 + (1 - |S_{11}|^2)(|S_{22}|^2 - |\Delta|^2)$$

(7.54)

$$= \delta + \alpha \beta$$

(7.55)

The expression (7.55) is based on the definitions

$$\alpha \overset{\Delta}{=} (1 - |S_{11}|^2)$$

(7.56)

$$\beta \overset{\Delta}{=} (|S_{22}|^2 - |\Delta|^2)$$

(7.57)

$$\delta \overset{\Delta}{=} |S_{12} S_{21}|^2$$

(7.58)
The original inequality, Eq. (7.50), written in terms of these new variables is given below:

\[ 2\sqrt{\delta} \sqrt{\alpha \beta + \delta} \leq (\alpha \beta + \delta) + \delta - \beta^2 \]  

(7.59)

By first squaring both sides and then canceling terms, Eq. (7.59) can be greatly simplified:

\[
4\delta(\alpha \beta + \delta) \leq \left[(\alpha \beta + 2\delta) - \beta^2\right]^2
\]

\[
4\delta(\alpha \beta + \delta) \leq (\alpha \beta + 2\delta)^2 - 2\beta^2(\alpha \beta + 2\delta) + \beta^4
\]

\[
4\delta(\alpha \beta + \delta) \leq (\alpha \beta)^2 + 4\delta(\alpha \beta) + 4\delta^2 - 2\beta^2(\alpha \beta + 2\delta) + \beta^4
\]

\[
0 \leq (\alpha \beta)^2 - 2\beta^2(\alpha \beta + 2\delta) + \beta^4
\]

\[
1 \leq \frac{(\alpha - \beta)^2}{4\delta}
\]

(7.60)

Taking the square root of Eq. (7.60) yields

\[
1 \leq \frac{\alpha - \beta}{2\sqrt{\delta}} = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = k
\]

(7.43)

Since the value of \( k \) is symmetrical on interchange of ports 1 and 2, the same result would occur with the generator port stability circle.

The second condition for unconditional stability, Eq. (7.44) can also be demonstrated based on the requirement that the \(|\Gamma_1| < 1\). The second term of the right-hand side of Eq. (7.17) can be modified by multiplying it by 1 \( (= S_{22}/S_{22}) \) and adding 0 \( (= S_{12}S_{21} - S_{12}S_{21}) \) to the numerator. This results in the following:

\[
|\Gamma_1| = \left| S_{11} + \frac{\Gamma_L S_{12}S_{21}S_{22} + (S_{12}S_{21} - S_{12}S_{21})}{(1 - \Gamma_LS_{22})S_{22}} \right|
\]

\[
= \frac{1}{|S_{22}|} \left| S_{11}S_{22}(1 - \Gamma_LS_{22}) - S_{12}S_{21}(1 - \Gamma_LS_{22}) + S_{12}S_{21} \right|
\]

\[
= \frac{1}{|S_{22}|} \left| \Delta + \frac{S_{12}S_{21}}{1 - \Gamma_LS_{22}} \right| < 1
\]

(7.61)

The complex quantity, \((1 - \Gamma_LS_{22})\) can be written in polar form as \((1 - |\Gamma_LS_{22}|e^{j\theta})\). Any passive load must lie within the unit circle \(|\Gamma_L| < 1\), so \(|\Gamma_L|\) is set to 1. As described in [5], the quantity

\[
\frac{1}{1 - |S_{22}|e^{j\theta}}
\]
Figure 7.8  Representation of the circle with $|\Gamma_L| = 1$.  

which appears in Eq. (7.61) is a circle, as pictured in Fig. 7.8, centered at

$$\frac{1}{2} \left[ \frac{1}{1 - |S_{22}|} + \frac{1}{1 + |S_{22}|} \right] = \frac{1}{1 - |S_{22}|^2}$$

and with radius

$$\frac{1}{2} \left[ \frac{1}{1 - |S_{22}|} - \frac{1}{1 + |S_{22}|} \right] = \frac{|S_{22}|}{1 - |S_{22}|^2}$$

Equation (7.61) is expressed in terms of this circle:

$$\frac{1}{|S_{22}|} \left| \Delta + \frac{S_{12}S_{21}}{1 - |S_{22}|^2} \right| + \frac{|S_{12}S_{21}|}{1 - |S_{22}|^2} < 1$$

(7.62)

The phase of the load is chosen so that it maximizes the left-hand side of Eq. (7.62). However, it must still obey the stated inequality. This means that Eq. (7.62) can be written as the sum of the two magnitudes without violating the inequality condition:

$$\frac{1}{|S_{22}|} \left| \Delta + \frac{S_{12}S_{21}}{1 - |S_{22}|^2} \right| + \frac{|S_{12}S_{21}|}{1 - |S_{22}|^2} < 1$$

$$0 < \frac{1}{|S_{22}|} \left| \Delta + \frac{S_{12}S_{21}}{1 - |S_{22}|^2} \right| < 1 - \frac{|S_{12}S_{21}|}{1 - |S_{22}|^2}$$

Comparison of the far right-hand side of this expression with 0 results in the following inequality:

$$1 - |S_{22}|^2 > |S_{12}S_{21}|$$

(7.63)

If the process had begun with the condition that $|\Gamma_o| < 1$, then the result would be the same as that of Eq. (7.63) with the 1’s and 2’s interchanged:

$$1 - |S_{11}|^2 > |S_{12}S_{21}|$$

(7.64)
When Eqs. (7.63) and (7.64) are added together,

\[ 2 - |S_{11}|^2 - |S_{22}|^2 > 2|S_{12}S_{21}|^2 \]  

(7.65)

However, from the definition of the determinate of the \( S \) parameter matrix,

\[ |\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < |S_{11}S_{22}| + |S_{12}S_{21}| \]  

(7.66)

When the term \(|S_{12}S_{21}|\) in Eq. (7.66) is replaced with something larger as given in Eq. (7.65), the inequality is still true:

\[ |\Delta| < |S_{11}S_{22}| + 1 - \frac{1}{2}(|S_{11}|^2 + |S_{22}|^2) \]

\[ |\Delta| < 1 - \frac{1}{2}(|S_{11}| - |S_{22}|)^2 < 1 \]  

(7.44)

An alternate, but equivalent set of requirements for stability, is [4]

\[ k > 1 \]  

(7.67)

and either

\[ B_1 > 0 \]  

(7.68)

or

\[ B_2 > 0 \]  

(7.69)

### 7.6.3 Stabilizing a Transistor Amplifier

There are a variety of approaches to stabilizing an amplifier. In Section 7.6.1 it was suggested that stability could be achieved from a potentially unstable transistor by making sure that the chosen amplifier terminating impedances remain inside the stable regions at all frequencies as determined by the stability circles.

Another method would be to load the amplifier with an additional shunt or series resistor on either the generator or load side. The resistor is incorporated as part of the two-port parameters of the transistor. If the condition for unconditional stability is achieved for this expanded transistor model, then optimization can be performed for the other circuit elements to achieve the desired gain and bandwidth. It is usually better to try loading the output side rather than the input side in order to minimize increasing the amplifier noise figure.

A third approach that is sometimes useful is to introduce an external feedback path that can neutralize the internal feedback of the transistor. The most widely used scheme is the shunt–shunt feedback circuit shown in Fig. 7.9. The \( y \) parameters for the composite circuit are simply the sum of the \( y \) parameters of the amplifier and feedback two-port circuits:

\[ [Y_c] = [Y_a] + [Y_f] \]  

(7.70)
To use this method, the transistor scattering parameters must be converted to admittance parameters (Appendix D). The $y$ parameters for a simple series admittance, $y_{fb}$ can be found from circuit theory (Fig. 7.10):

\[
\begin{align*}
y_{11} &= y_{22} = \lim_{v_2 \to 0} \frac{i_1}{v_1} = y_{fb} \quad \text{(7.71)} \\
y_{12} &= y_{21} = \lim_{v_2 \to 0} \frac{i_2}{v_1} = -y_{fb} \quad \text{(7.72)}
\end{align*}
\]

Consequently the composite $y$ parameters are

\[
\begin{align*}
y_{11c} &= y_{11a} + y_{11f} = y_{11a} + y_{fb} \quad \text{(7.73)} \\
y_{12c} &= y_{12a} + y_{12f} = y_{12a} - y_{fb} \quad \text{(7.74)} \\
y_{21c} &= y_{21a} + y_{21f} = y_{21a} - y_{fb} \quad \text{(7.75)} \\
y_{22c} &= y_{22a} + y_{22f} = y_{22a} + y_{fb} \quad \text{(7.76)}
\end{align*}
\]

If $y_{12c}$ could be made to be zero, then $S_{12c}$ would also be zero and unconditional stability could be achieved:

\[
g_{12a} + jb_{12a} = g_{fb} + jb_{fb} \quad \text{(7.77)}
\]

Since the circuit parameter $g_{12a} < 0$, the value $g_{fb} < 0$ must be true also. Since it is not possible to have a negative passive conductance, complete removal of
the internal feedback is not possible. However, the susceptance, \( b_{12a} \), can be canceled by a passive external feedback susceptance. Although total removal of \( y_{12a} \) cannot be achieved, yet progress toward stabilizing the amplifier can often be achieved. There is no guarantee that neutralization will provide a composite \( y \) matrix that is unconditionally stable. In addition neutralization of the feedback susceptance occurs at only one frequency.

As an example consider a transistor to have the following \( S \) parameters at a given frequency:

\[
\begin{align*}
S_{11a} &= 0.73 \angle -102^\circ \\
S_{21a} &= 2.21 \angle 104^\circ \\
S_{12a} &= 0.10 \angle 48^\circ \\
S_{22a} &= 0.47 \angle -48^\circ 
\end{align*}
\]

(7.78)

For this transistor, \( k = 0.752 \) and \( |\Delta| = 0.294 \) as found from Eqs. (7.43) and (7.44). Conversion of Eq. (7.78) to \( y \) parameters gives

\[
\begin{align*}
y_{11a} &= 5.5307 \cdot 10^{-3} + j1.9049 \cdot 10^{-2} \ S \\
y_{12a} &= 3.9086 \cdot 10^{-2} - j2.3092 \cdot 10^{-3} \ S \\
y_{21a} &= 4.7114 \cdot 10^{-2} - j2.1376 \cdot 10^{-2} \ S \\
y_{22a} &= 5.4445 \cdot 10^{-3} + j5.1841 \cdot 10^{-3} \ S.
\end{align*}
\]

(7.79)

Nothing can be done about \( g_{12a} \), but \( b_{12a} \) can be removed by setting \( b_{fb} = b_{12a} = -2.3092 \). The composite admittance matrix becomes

\[
\begin{align*}
y_{11c} &= 5.5307 \cdot 10^{-3} + j1.6739 \cdot 10^{-2} \ S \\
y_{12c} &= 3.9086 \cdot 10^{-4} - j(0) \ S \\
y_{21c} &= 4.7114 \cdot 10^{-2} - j1.9067 \cdot 10^{-2} \ S \\
y_{22c} &= 5.4445 \cdot 10^{-3} + j2.8750 \cdot 10^{-3} \ S.
\end{align*}
\]

(7.80)

The composite scattering parameters can now be found and the stability factor calculated yielding \( k = 2.067 \) and \( |\Delta| = 0.4037 \). The transistor with the feedback circuit is unconditionally stable at the given frequency. This stability has been achieved by adding inductive susceptance in shunt with the transistor input and output ports.

Broadband stability can be achieved by replacing the feedback inductor with an inductor and resistor as shown in Fig. 7.11. A starting value for the inductor can be found as described for the single frequency analysis. The resistor is typically in the 200 to 800 \( \Omega \) range, but optimum values for \( R \) and \( L \) are best found by computer optimization.
7.7 CLASS A POWER AMPLIFIERS

Class A amplifiers, whether for small signal or large signal operation, are intended to amplify the incoming signal in a linear fashion. This type of amplifier will not introduce significant distortion in the amplitude and phase of the signal. A linear class A power amplifier will introduce low harmonic frequency components and low intermodulation distortion (IMD). An example of intermodulation distortion can be described in terms of a double sideband suppressed carrier wave, which is represented as

\[
\frac{V}{2} \cos(\omega_c + \omega_m)t + \frac{V}{2} \cos(\omega_c - \omega_m)t
\]  

(7.81)

where \(\omega_c\) is the high-frequency carrier frequency and \(\omega_m\) is the low-frequency modulation frequency. Intermodulation distortion would result in frequencies at \(\omega_c \pm n\omega_m\), and harmonic distortion would cause frequency generation at \(k\omega_c \pm n\omega_m\). The later harmonic distortion can usually be filtered out, but the intermodulation distortion is more difficult to handle because the distortion frequencies are near if not actually inside the system pass band. Clearly, this distortion in a class A amplifier is a greater problem for power amplifiers than for small signal amplifiers. Reduction of IMD depends on efficient power combining methods and careful design of the transistors themselves.

A transistor acting in the class A mode remains in its active state throughout the complete cycle of the signal. Two examples of common emitter class A amplifiers are shown in Fig. 7.12. The maximum efficiency of the class A amplifier in Fig. 7.12a has been shown to be 25%, for example [6]. However, if an RF coil can be used in the collector (Fig. 7.12b), the efficiency can be increased to almost 50%. This can be shown by recognizing first that there is no ac current flow in the bias source and no dc current flow in the load, \(R_L\). The total current flowing in the transistor collector is

\[
i_c = I_Q - I_o \sin \omega t
\]  

(7.82)

and the total collector voltage is

\[
V_{cc} = V_{CC} + V_o \sin \omega t
\]  

(7.83)
The quiescent current, $I_Q$, and the output current, $I_o$, is defined in Fig. 7.13. When the load is drawing the maximum instantaneous power,

$$I_{o,\text{max}} = I_Q = I_{dc}. \quad (7.84)$$

At this point the maximum output voltage is

$$V_{o,\text{max}} = I_{o,\text{max}}R_L \quad (7.85)$$

and

$$|V_{o,\text{max}}| = V_{CC} = I_{o,\text{max}}R_L \quad (7.86)$$

The dc power source supplies

$$P_{dc} = I_{dc}V_{CC} = \frac{V_{CC}^2}{R_L} \quad (7.87)$$
The maximum average power delivered to the load can now be written in terms of the supply voltage:

$$P_o = \frac{|V_{o,\text{max}}|^2}{2R_L} = \frac{V_{CC}^2}{2R_L}$$

(7.88)

If the RF input power is $P_i$, the power added efficiency is

$$\eta = \frac{P_o - P_i}{P_{dc}}$$

(7.89)

For high-gain amplifiers, $P_i \ll P_o$ and the maximum efficiency is $\eta \approx \frac{1}{2}$. However, it should be noted that many times high-power amplifiers do not have high gain, so the power added efficiency given by Eq. (7.89) offers a more useful quality factor for a transistor than if $P_i$ were neglected.

### 7.8 POWER COMBINING OF POWER AMPLIFIERS

Design of power FET amplifiers requires use of large gate periphery devices. However, eventually, the large the gate periphery causes other problems such as impedance matching especially at RF and microwave frequencies. Bandwidth improvement can be obtained by combining several transistors, often on a single chip. An example of combining two transistors is shown in Fig. 7.14 [7,8]. The separation of the transistors may induce odd-order oscillations in the circuit, even if the stability factor of the individual transistors (even-order stability) indicate they are stable. This odd-order instability can be controlled by adding $R_{\text{odd}}$ between the two drains to damp out such oscillations. This resistor is typically less than 400 $\Omega$. Symmetry indicates no power dissipation when the outputs of the two transistors are equal and in phase. An example of a four transistor combining circuit is shown in Fig. 7.15, which now includes resistors $R_{\text{odd1}}$ and $R_{\text{odd2}}$ to help suppress odd-order oscillations.

![FIGURE 7.14](image_url)  
*Power combining two transistors [7,8].*
FIGURE 7.15  Power combining four transistors [7,8].

PROBLEMS

7.1 Using the flow graph reduction method, verify the reflection coefficient found in Eq. (7.17).

7.2 The measured scattering parameters of a transistor in an amplifier circuit are found to be the following:

\[
\begin{align*}
|S_{11}| & \quad \angle S_{11} & \quad |S_{21}| & \quad \angle S_{21} & \quad |S_{12}| & \quad \angle S_{12} & \quad |S_{22}| & \quad \angle S_{22} \\
0.85 & \quad -32 & \quad 3.8 & \quad -145 & \quad 0.04 & \quad 74 & \quad 0.92 & \quad -15
\end{align*}
\]

(a) Determine the stability factor, \( k \), for this transistor.

(b) Determine the \( y \) parameters for this circuit.

(c) Determine the circuit that would neutralize (almost unilateralize) the circuit. While this procedure does not guarantee stability in all cases, it always helps lead toward greater stability.

(d) Determine the new scattering parameters for the neutralized circuit.

(e) Determine the generator and load impedances that would give maximum transducer power gain (not unilateral power gain).

(f) What is the value for the maximum transducer power gain.

7.3 Determine the transfer function for the flow graph in Fig. 7.16.

7.4 A certain transistor has the following \( S \) parameters:

\[
S_{11} = 1.2, \quad S_{21} = 4.0, \quad S_{12} = 0, \quad S_{22} = 0.9
\]

Determine whether this transistor is unconditionally stable.
REFERENCES


FIGURE 7.16  Flow graph for Problem 7.3.