COMPRESSED SENSING FRAMEWORK FOR EEG COMPRESSION

Selin Aviyente

Department of Electrical and Computer Engineering,
Michigan State University, East Lansing, MI, 48824

ABSTRACT
Many applications in signal processing require the efficient representation and processing of data. The traditional approach to efficient signal representation is compression. In recent years, there has been a new approach to compression at the sensing level. Compressed sensing (CS) is an emerging field which is based on the revelation that a small collection of linear projections of a sparse signal contains enough information for reconstruction. In this paper, we propose an application of compressed sensing in the field of biomedical signal processing, particularly electroencephalogram (EEG) collection and storage. A compressed sensing framework is introduced for efficient representation of multichannel, multiple trial EEG data. The proposed framework is based on the revelation that EEG signals are sparse in a Gabor frame. The sparsity of EEG signals in a Gabor frame is utilized for compressed sensing of these signals. A simultaneous orthogonal matching pursuit algorithm is shown to be effective in the joint recovery of the original multiple trial EEG signals from a small number of projections.

Index Terms— Biomedical signal processing, Electroencephalography, Signal Reconstruction, Sampling

1. INTRODUCTION
Electroencephalogram (EEG) is a commonly used brain imaging method with high temporal resolution with applications in neurology and psychology [7]. In a typical clinical setting, EEG signals are collected over multiple channels, ranging from 64 to 128 channels, and with multiple trials, i.e. the same stimulus may be repeated multiple times to obtain high signal-to-noise ratio estimates of the evoked response. One of the biggest challenges with EEG data collection and analysis is the amount of data that needs to be stored and processed. EEG activity collected from a single subject can correspond to 2-3 hours of data. Therefore, there is a growing need for data reduction or compression methods that can efficiently compress the EEG data into a few number of samples. An efficient compression of the EEG data would reduce the amount of data that needs to be stored and would also allow for the fast wireless transmission of the collected EEG data in a clinical setting.

In recent years, a new paradigm, named Compressed Sensing (CS), has been proposed for signal sensing and compression [1, 2]. CS builds on the revelation that a signal having a sparse representation in one basis can be recovered from a small number of projections onto a second basis that is incoherent with the first. This revelation has promising implications for applications for signal acquisition and compression. With no a priori knowledge of a signal’s structure, a sensor node could simultaneously acquire and compress the signal, preserving the critical information. Some recent applications of this theory include the single pixel camera [3] and compressed sensing for rapid MR imaging [4].

In this paper, we use the framework of CS for EEG signal compression and reconstruction. We first show that EEG signals are sparse in a Gabor frame based on an empirical study using a large set of EEG data. Next, we apply the CS framework to compress single-trial EEG recordings using a few number of projections onto an i.i.d. Gaussian basis. The reconstruction of the actual EEG signal from these projections is achieved using orthogonal matching pursuit algorithm as proposed in [5]. After the sparsity of individual EEG signals is established, we extend the CS framework to joint recovery of multiple signals using recent results in distributed compressed sensing [6]. The joint sparsity of multiple EEG signals is shown and a simultaneous orthogonal matching pursuit algorithm is used to reconstruct multiple recordings simultaneously.

2. BACKGROUND ON COMPRESSED SENSING
In this section, we will give a brief background on compressed sensing theory. Given a signal $x$ and a dictionary $\Psi$, $x$ is said to be sparse if $x$ can be well approximated by a linear combination of a small set of vectors from $\Psi$, i.e. $x \approx \sum_{i=1}^{K} a_{n_i} \psi_{n_i}$, where $K << N$. The CS theory states that it is possible to construct an $M \times N$ measurement matrix $\Phi$, where $M << N$, and reconstruct $x$ from the measurements $y = \Phi x$ [1]. It has been shown that the signal can be recovered from its measurements when the measurement matrix is incoherent with the dictionary that the signal is sparse over. Some choices for the measurement matrix, $\Phi$, are a random matrix with i.i.d. Gaussian entries or the Bernoulli ($\pm 1$) matrix. Using such a matrix $\Phi$ of size $cK \times N$, where $c$ is an
oversampling factor, it was shown that it is possible, with high probability, to recover any signal that is K-sparse in the dictionary \( \Psi \) from its projection onto \( \Phi \).

Several algorithms have been proposed for recovering \( x \) from the measurements \( y \) by solving the following optimization problem:

\[
\arg\min_{\mathbf{a}} \| \mathbf{a} \|_1 \text{ subject to } \Phi \mathbf{a} = \mathbf{y}
\]  

The canonical approach uses linear programming, but has high computational complexity. Greedy algorithms such the matching pursuit (MP) and orthogonal matching pursuit (OMP) require fewer computations at the expense of slightly more measurements [5].

3. SPARSITY OF EEG SIGNALS

In previous work, it has been shown that EEG signals can be efficiently parameterized in terms of the elements of a Gabor dictionary [7, 8, 9]. The Gabor dictionary has been commonly used to extract different time-frequency features from the EEG signals for event characterization and classification as in [8]. However, the sparsity of these signals have not been officially shown on large sets of data.

Given a Gabor dictionary with elements,

\[
g_i(n; n_0, f_0, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(n - n_0)^2}{2\sigma^2}\right) \exp(-j2\pi f_0 n),
\]

where \( n_0 \) and \( f_0 \) are the time and frequency centers of the Gabor logon, respectively, and \( \sigma \) is the spread of the logon, previous work has shown that any EEG signal can be written as \( x(n) \approx \sum_{i=1}^{K} a_i g_i(n) \) where \( K \) is much less than the number of samples \( N \).

In this paper, we investigate the sparsity level of EEG signals based on the analysis of 750 sample long EEG recordings of a control subject performing a continuous performance task (CPT). The same task is repeated 116 times to produce 116 EEG trials. First, we approximate each EEG recording using the elements of the Gabor dictionary over the given time range and a frequency range of 1-50 Hz. The normalized mean squared error between the approximated signal, \( \hat{x} = \sum_{i=1}^{K} a_i g_i(n) \) and the original signal is computed and averaged over 116 trials for different values of \( K \). Fig. 1 (a) shows the average normalized mean squared error (NMSE) versus the sparsity level. It can be seen that the NMSE is low and rolls-off fast as \( K \) increases. Fig. 1(b) shows an example EEG recording and its reconstruction using 10 elements of the Gabor dictionary.

Using the fact that EEG recordings are sparse, one can use the CS framework to reconstruct these signals from \( cK \) measurements. In this paper, the projection matrix, \( \Phi \), is chosen as i.i.d. Gaussian random vectors. The measurement vector \( y = \Phi x = \Phi \Psi a \) is sparse in the dictionary \( \Phi \Psi \), since \( a \) is a sparse coefficient vector. Therefore, reconstructing \( x \) boils down to solving for the sparse vector \( a \) and then using the relationship \( x = \Psi a \). The sparse vector \( a \) can be found by using the OMP algorithm discussed in [5]. Fig. 2 shows the average NMSE in reconstruction versus the number of measurements. From this figure, it can be seen that one can reconstruct the EEG signal with a small amount of error by storing 80 measurements instead of the 750 samples in the original recording.

3.1. Joint Sparsity of EEG Signals

Most of the existing work in EEG signal analysis focuses on the representation of single-trial EEG signals on a given dictionary or the representation of the sample average. The results of the previous section show that individual EEG signals are sparse in a Gabor frame and can be reconstructed...
from a small number of projections onto an incoherent basis. Since EEG signals are usually recorded over multiple trials and electrodes, this projection and the following reconstruction has to be done on each signal, individually. This approach is not practical as it would become computationally expensive to construct different projection matrices for each signal and to reconstruct them separately. Therefore, it is important to investigate the existence of a basis or dictionary that can efficiently represent a collection of EEG signals since the joint sampling and reconstruction of EEG signals would prove to be more beneficial in reducing the number of samples that are collected compared to the individual sampling.

In this section, we assume the following joint sparsity model for a collection of EEG signals, $x_j, j \in \{1, 2, \ldots, J\}$, where $J = 116$ in our case.

$$x_j = \Psi a_j,$$  \hspace{1cm} (3)  

where each $a_j$ is supported on the same subset of the dictionary. Hence, all signals are $K$-sparse and are reconstructed from the same $K$ elements of $\Psi$, but with arbitrarily different coefficients. This is a reasonable assumption for EEG recordings since the different sensors measure the same activity with phase difference and attenuation caused by the signal propagation in the brain.

In this paper, we consider the 116 trials of EEG activity recorded during a continuous performance task. We assume that the different trials, similar to the different electrodes, measure the same underlying activity with phase resettings. We investigate the joint sparsity of 116 trials over the Gabor dictionary as illustrated in Fig. 3. From this figure, it can be seen that 40 elements from the Gabor dictionary result in an average NMSE that is comparable to the average NMSE obtained by choosing 3 dictionary elements per trial (see Fig. 1 (a)). This means that by using the joint sparsity we can reduce $K$ from $3 \times 116 = 348$ to 40 and achieve the same NMSE.

Since the EEG signals are jointly sparse, we can jointly reconstruct the signals from their individual measurements using the distributed compressed sensing-simultaneous OMP algorithm discussed in [6]. Fig. 4 illustrates the average NMSE in the joint reconstruction of the EEG signals versus the number of measurements. It can be seen that the error rates in Fig. 4 are slightly higher than the ones found for the individual trial reconstruction in Fig. 2 since the same set of elements from the Gabor dictionary are used to reconstruct all of the EEG recordings.

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Fig. 3. Normalized Mean Square Error in reconstruction versus the number of dictionary elements used

Fig. 4. Normalized Mean Square Error in reconstruction versus the number of measurements

5. CONCLUSIONS

In this paper, we adapted the compressed sensing framework for the compression and representation of EEG signals. First, we showed the sparsity of individual EEG recordings in a Gabor dictionary and used that information to reduce the number of measurements required to reconstruct the EEG signal. We then extended this to a joint sparsity framework to further reduce the number of measurements. The proposed work can be extended by modifying the Gabor frame to a chirped Gabor dictionary to further reduce the sparsity level and the corresponding number of measurements. Moreover, the quality of the reconstructed signals can be evaluated criteria other than mean-squared error such as qualitative measures that describe the quality of diagnosis.
6. REFERENCES


