Signal Quantization Effects on Acquisition Process of GPS Receiver: the Analysis and Simulation

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Abstract

Signal quantization levels can have significant impact on the performance of a GPS receiver, especially in the presence of the interference. In this paper, signal quantization model for estimating interference suppression of the correlator and evaluating the performance of GPS receiver is presented. With the proposed model, performance of the receiver in acquisition process is demonstrated by showing the quantization effects on SNR degradation and weak signal detection. Test results show the limits of the low-SNR acquisition of the receiver and the better performance GPS receiver will achieve with a proper set quantization method.

1. Introduction

Global Positioning System receivers need to acquire and track the pseudo-random number (PRN) code to make use of the GPS signals from a particular satellite. The acquisition process can get an estimated PRN code phase and the carrier Doppler shift via a two dimension search pattern. The qualities of the result which is used to initial tracking may represent the sensitivity of a receiver.

With an increasing demand on GPS receiver performance in varied applications, the ability to acquire and track weak signals in different kinds of interference is of great importance. Lots of researchers made contributions to this work in the software receiver design. For example, Psiaki claimed block acquisition [1] of GPS signal for weak GPS signal, and bit-wise algorithm [2]. In order to increase the S/N, long coherent integration time is used. However, since the code phase transition of navigation data takes places every 20 ms, the data length to be used is limited. Without any aid from other source, the limit for coherent integration is about 40ms [3]. A higher performance requirement may result in a consideration about the relationship between hardware and software design.

The present paper concentrates on the problem of signal quantization effects on acquisition process of receiver. Exclusively, it deals with GPS L1 frequency C/A code. The paper goes on to show the relationship between probabilities of detection, false alarm, threshold, integration time in deferent quantization.

2. Acquisition process

Acquisition is a coarse synchronization process which produces the estimates of PRN code offset and the carrier Doppler. The whole process is a two dimension search during which the local replica code and carrier are aligned with the received signal. If both the code and carrier Doppler match the incident signal, the signal is de-spread and a carrier signal is recovered. The following figure is the conventional acquisition process of the GPS receiver.

![Figure 1 acquisition scheme](image)

In this acquisition scheme, the In-phase (I) and Quadrature (Q) components are generated after the correlators. Both I and Q data are accumulated for several code periods and then the sum is squared. With a threshold, the envelope of $\sqrt{I^2+Q^2}$ is used to detect whether the code phase and carrier Doppler match the incident signal.

Besides the acquisition search pattern is two dimensional. In the code phase direction, the search follows the range direction from early to late. In the Doppler bin direction, the search pattern typically starts with the mean value of the Doppler uncertainties.
Then it goes from one Doppler bin to another until all 3-sigma Doppler uncertainties has been searched.

3. Signal quantization model and its effects on the correlator

The received C/A signal from one satellite has coherently down-converted to baseband along with the Gaussian noise and sinewave interference. The interference varies slowly with an angle while its amplitude is constant K. In this example, we assume the interference is K. Then the received signal \( r(t) \) is:

\[
   r(t) = Ap(t) + K + n(t)
\]

(1)

Where \( p(t) \) is the PRN code, the interference has no PN modulation and its amplitude K is constant, \( n(t) \) is noise.

With the output of m-bit quantization \( Q_m[r(t)] \), output of correlator is:

\[
   R_p = p(t)Q_m[r(t)] = p(t)Q_m[Ap(t) + K + n(t)]
\]

(2)

\( Q_m[r(t)] \) has a probability of 0.5 to have mean value \( K+A \), variance \( \sigma^2 \), and a probability of 0.5 to have mean value \( K-A \), variance \( \sigma^2 \), since the PRN code \( p(t) \) has a probability of 0.5 to be 1 or -1.

\[
   Q[r(t)] \sim \frac{1}{2} \text{Normal}(K+A, \sigma^2) + \frac{1}{2} \text{Normal}(K-A, \sigma^2)
\]

In 2-bit quantization, \( \Delta \) is quantizing interval, the output levels are \( \{-3,-1,1,3\} \). Using the relationship \( F(y)=0.5[\text{erf}(y/(\sqrt{2}\sigma))]+1 \) between error function and Gaussian distribution, the expected correlator output can be simplified as follows:

\[
   E[c(t)] = \frac{1}{4} \left[ \text{erf} \left( \frac{\Delta-(K+1)}{\sigma \Delta} \right) - \text{erf} \left( \frac{\Delta-(K-1)}{\sigma \Delta} \right) + \text{erf} \left( \frac{\Delta+(K-1)}{\sigma \Delta} \right) - \text{erf} \left( \frac{\Delta+(K+1)}{\sigma \Delta} \right) \right]
\]

\[
   + \frac{3}{4} \left[ \text{erf} \left( \frac{\Delta-(K-1)}{\sigma \Delta} \right) - 1 + \text{erf} \left( \frac{\Delta+(K-1)}{\sigma \Delta} \right) \right]
\]

\[
   - \frac{1}{4} \left[ \text{erf} \left( \frac{\Delta+(K+1)}{\sigma \Delta} \right) - \text{erf} \left( \frac{\Delta-(K+1)}{\sigma \Delta} \right) + \text{erf} \left( \frac{\Delta-(K-1)}{\sigma \Delta} \right) - \text{erf} \left( \frac{\Delta+(K-1)}{\sigma \Delta} \right) \right]
\]

\[
   - \frac{3}{4} \left[ \text{erf} \left( \frac{\Delta+(K-1)}{\sigma \Delta} \right) - \text{erf} \left( \frac{\Delta-(K-1)}{\sigma \Delta} \right) \right]
\]

In the similar way, we can obtain the \( E[c^2(t)] \).

The output Signal to Noise Ratio of the correlator is:

\[
   (SNR) = \frac{E[c(t)]^2}{E[c^2(t)]} - E[c(t)]^2
\]

(3)

By using the equation (4), output SNR of the correlator can be calculated.

In the case of 1-bit quantization, we can just set \( \Delta=0 \) in the equation (3). The following figures show the relationship between SNR degradation of the correlator and the interference amplitude K under different quantization methods.

![Figure 2](SNR versus Interference level, 1-bit quantization)

![Figure 3](SNR versus Quantizing interval, 2-bit quantization)

![Figure 4](SNR versus interference and quantizing intervals)

It is clearly showed in Figure 2, 3 that 1-bit quantization can cause a rapid SNR degradation of the correlator output with increasing interference amplitudes. When K is 20 the SNR degradation is about 20dB. However, in 2-bit quantization, the SNR degradation of the correlator output under same levels of interference is greatly mitigated, especially when the quantizing intervals are properly set. For example, in 2-bit cases proper intervals can bring a nearly 12dB degradation.

In Figure 4, we can see that with a proper set quantizing interval, 2-bit quantization can greatly suppress the high levels of interference.
In the acquisition process, the input signal in several units of 1ms is squared after the correlation in order to find the initial code phase which can be used to strip off the C/A code. Since the quantization influences the output SNR of the correlator, it indirectly causes a different type of SNR loss in the squared parts.

Consider that the signal amplitude is $A$, the noise is $n$. The signal $x$, corresponding S/N in dB are:

$$x = A + n$$

$$\frac{S}{N} = 20 \log \left( \frac{A}{\sigma_n} \right)$$

Where $\sigma_n$ is the standard deviation of the noise, when $x$ is squared, we can get:

$$x^2 = A^2 + (2nA + n^2)$$

In the above equation, the front part is considered as the squared signal amplitude while the back part is considered as the noise amplitude. Corresponding S/N in dB is:

$$\frac{S}{N} = -10 \log (4 + 3 \times 10^{-0.15/10})$$

(4)

Figure 5 shows the SNR degradation through the squared parts. With equation (3) and (4), the degradation of the SNR according to different interference suppression through the squared parts can be obtained.

4. Effects on receiver performances

As is described above, GPS signal acquisition is a search process while it is of great importance to remember that the C/A code autocorrelation and cross-correlation side-lobes can cause false signal detections when the energy of the side-lobes are strong enough. One possible measure to enhance the detection performance is that a combination of both increased dwell time, higher level of detection threshold used in the initial step and a decrease of both in the subsequent searches. The drawback for this method is increased search time while $C/N_0$ is relatively low.

The rule of thumb follows is to estimate and compute the envelope $\sqrt{I^2 + Q^2}$ in the dwell time $T$. A threshold $\gamma$ is needed which be compared with the envelope to determine whether the SV signal is absent ($H_0$) or present ($H_1$). The detection of signal is a statistic process.

It can be assumed that I and Q have a Gaussian distribution and the envelope is formed by $\sqrt{I^2 + Q^2}$.Composite the following hypotheses:

$$H_0 : E(Q) = E(I) = 0, \text{Var}(Q) = \text{Var}(I) = \frac{N_0}{2}$$

$$H_1 : E(Q|\theta) = E(I|\theta) = \sqrt{E^2 \cos \theta}, E(I|\theta) = \sqrt{E^2 \sin \theta}, \text{Var}(Q) = \text{Var}(I) = \frac{N_0}{2}$$

Then we can easily find that:

$$P_{fa} = \Pr[I > \gamma | H_0] = \int \int (2\pi \frac{N_0}{2})^{-1} \exp(-\frac{I^2 + Q^2}{N_0})dIdQ$$

Changing it to the polar coordinate then:

$$P_{fa} = \exp(-\frac{\gamma}{N_0})$$

(5)

So the threshold can be determined by a desired single trial of probability of false alarm and the 1-sigma noise power:

$$\gamma = \sigma_n \sqrt{-2 \ln P_{fa}}$$

(6)

Similarly, detection probability is:

$$P_d = \int \int (2\pi \frac{N_0}{2})^{-1} \exp(-\frac{(Q-\sqrt{E\cos \theta})^2 + (I-\sqrt{E\sin \theta})^2}{N_0})dIdQ$$

Letting $Q = R \cos \beta, I = R \sin \beta, z = \sqrt{2/N_0}R$ and we can obtain that:

$$P_d = \int \int \frac{z \exp(-\frac{z^2 + d^2}{2})I_0(az)dz}{d}$$

As we expected, $P_d$ has nothing to do with $\theta$ and variable $z$ follows a Ricean distribution.

Finally, if Marcum’s Q function is used to find relation between detection probability and false alarm probability, we can get:

$$P_d = Q(d, \sqrt{-2 \ln P_{fa}})$$

(7)

$$(Q(a,b)) = \int \int \frac{z \exp(-\frac{z^2 + d^2}{2})I_0(az)dz}{d}$$

Equation (6) shows the relationship between false alarm probability and threshold of the detection. And with equation (7), the probability of detection can be obtained with a given false alarm probability and bit SNR. Consider the different characteristics in various quantization methods, we can get:
5. Conclusions

This paper has analyzed the sensitivity of the GPS receiver and a serial of SNR degradations in the present of interference under different signal quantization. These results are well suited for the improving of hardware and software GPS receiver design for the tracking and acquisition of weak signal. Multi-bit quantization with a proper set quantizing interval will sufficiently suppress different levels of the interference. Respectively, it will also enhance the performances of receiver in lower $C/N_0$. Moreover it proves that for a commonly used stand-alone GPS receiver, the limit $C/N_0$ is nearly 25dB. Results based on simulations are presented to verify the validity of the analysis in different quantization cases.

6. References

[1] Mark L. Psiaki, Block Acquisition of Weak GPS signal in a Software Receiver [J], ION 2001, pp2838–2850