Research of Weak GPS Signal Acquisition Algorithm

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Abstract—GPS has been broadly applied in every fields of our social life, since the environment around us has been becoming more and more complicated, satellite navigation positioning technology with low SNR gains more and more attention. However, the current GPS signal structure and signal power levels are barely sufficient for indoor applications. Errors due to multi-path and noise associated with weak indoor signals limit the accuracy and availability of GPS in difficult indoor environments. This paper present an overview of GPS positioning technologies based on indoor environments. An adapted acquisition algorithm based on differential correlation is introduced. Analyzes two common methods for improving the SNR of the test statistic: coherent integration (COH) and non-coherent integration (NCH), then points out their disadvantage when used in low-SNR circumstance. By means of math model, the adapted acquisition algorithm based on differential correlation (DF) is introduced in detail. With the help of simulation, the simulation results indicate that this algorithm can acquire GPS signal with low SNR using the shorter time.

I. INTRODUCTION

Global Positioning System (GPS) receivers must acquire and track the pseudorandom codes and carrier signals from several GPS satellites. In order to track and decode the information in GPS signal, an acquisition method must first be used to detect the presence of the signal. Once the signal is detected, the necessary parameters must be obtained and passed to a tracking program. Information such as the navigation data can be obtained from the tracking program. The navigation mode estimates the PRN code phase and the carrier Doppler shift via a search process. These quantities are then used to initiate tracking, which continuously updates them.

GPS signal acquisition and tracking are notoriously difficult when signals are weak. The general architecture of acquisition units for weak GPS signals can be categorized as either software-based, where the time domain code phase ambiguity is tested in parallel on a DSP via a single Fast Fourier Transform (FFT), or hardware-based, where multiple correlators are simultaneously dedicated to the hypothesis test in several dwells. For both types, an acquisition search progresses until a decision variable in one code/frequency bin is greater than a predefined threshold. This predefined detection threshold, which is a function of the noise floor and required detection statistics, in part determines the receiver’s acquisition performance. Most commercial receivers can use a signal only if its SNR is greater than about 35-37 dB-Hz [1,2]. The signal sensitivity for an indoor signal is normally 10-30 dB lower than that of a line of sight signal. So most GPS signal acquisition techniques, which are considered to be adequate on positioning capability in outdoor environment are not satisfactory for application with weak signals.

There is a great amount of attention on exploring the techniques for location estimation in weak signal environment. It has been investigated in [3] that the attenuation and interference to the received GPS signal are severe in stringent circumstance. Different types of technique have been proposed to overcome the problem, which can be categorized into the Assisted-GPS methods, COH and NCH. The A-GPS methods primarily utilize the messages coming from the cell-based systems to assist the acquisition of the received GPS signal. In order to improve signal sensitivity, the COH acquisition technique maximize the COH period. However, for too weak signal, processing gain obtained through COH may not be sufficient for detection. In addition, the existence of navigation bit transition limits the COH period to 20 ms, normally 10 ms without the external aiding, using the current L1 signal, although the block acquisition scheme proposed in alleviates the navigation bit-transition problem using the COH method. However, the long COH time increases the number of frequency search cells, which in turn increases the time to first fix. Meanwhile, the COH time is also limited by the user dynamics and the quality of the front-end oscillator. Using NCH technique, integration can be extended for much longer periods but the processing gain suffers severely because of the squaring loss is incurred. Squaring loss increases as pre-squaring SNR decreases, leading to a fundamental limit to the sensitivity of this technique [4].

In this paper, an adapted acquisition algorithm based on DF is proposed for GPS acquisition under weak signal
environment. The main concept of the DF scheme is to use
differentially-coherent combining instead of non-coherent
combining of the COH results for the acquisition of GPS
signals. Although this method becomes particularly useful
within this context, as we will explain, it has not been
considered for this purpose so far.

There are two reasons why differentially-coherent
combining becomes particularly attractive within the context
of GPS signal acquisition. First, differentially-coherent
combining can only work if the signals to be combined are of
the same polarity. With BPSK data modulation this is
normally not the case. However, the data rate of the GPS
navigation message is only 50Hz while the C/A code period
is 1ms. That means that at the output of the C/A code
correlator a sign reversal may occur once every 20
correlation results at most. Thus data modulation will, in
the worst case, only lead to a very small loss of about 1 dB,
which means that assistance information may not be required
at all.

Secondly, the frequency offset of the input signal due to
the Doppler effect translates into a constant phase rotation of
the differentially combined correlation results. Thus, the
Doppler offset, which also has to be determined by the
acquisition unit, can easily be estimated from the phase of
the differentially combined correlation results.

This paper first analyzes two common methods for
improving the SNR of the test statistic: COH and NCH, then
points out their disadvantage when used in low-SNR
circumstance. The second section mainly introduced an
adapted acquisition algorithm based on DF.

II. COH AND NCH

In general, in order to enhance the signal sensitivity, the
COH is preferred where the correlation values are
algebraically added over a certain period. For a stand-alone
GPS receiver, during acquisition, navigation data bit
transitions are not known. The COH may actually cancel the
correlation value. Therefore, the integration period is limited
to 10 ms. If data aiding is available, the integration time may
be extended to 20 ms or beyond. However, the long COH
time increases the number of frequency search cells, which
in turn, increases the time to first fix. Meanwhile, the COH
time is also limited by the user dynamics and the quality of
the front-end oscillator. Normally, the NCH is implemented
after a short period of the COH, which is not influenced by
data bit transitions and the quality of the oscillator. Equation
(1) and equation (2) below show the integration schemes of
the COH and NCH, respectively:

\[ S_\phi = \sum_{k=1}^{T_{COH}} I_k \quad S_Q = \sum_{k=1}^{T_{COH}} Q_k \]

(1)

\[ S_{\phi 2} = \sum_{k=1}^{N} \sqrt{I_k^2 + Q_k^2} \]

(2)

Where,

\[ S_\phi: \text{ In-phase integration value} \]

\[ S_{\phi 2}: \text{ Quadrature-phase integration value} \]

\[ S_{\omega c}: \text{ NCH value} \]

\[ I_k: \text{ 1ms in-phase accumulator output} \]

\[ Q_k: \text{ 1ms Quadrature-phase accumulator output} \]

\[ T_{COH}: \text{ Number of coherent summations} \]

\[ M: \text{ Number of non-coherent summations} \]

From equation (2), it can be seen that NCH requires
squaring of the accumulator outputs. As a result, the noise is
squared and can not be cancelled by summation. This
non-linear process induces the so-called squaring loss. The
squaring loss depends on the SNR after COH and before
NCH. The higher the SNR before NCH, the lower the
squaring loss.

III. AN ADAPTED ACQUISITION ALGORITHM BASED ON DF

The following is a model of GPS signal that has been
used in this study

\[ r(t) = AD(t)c(t)\cos(\omega t) + n(t) \]

(3)

In this model, the constant A is the signal amplitude. The
function \( D(t) \) is the GPS data stream. The function \( c(t) \) is
the C/A PRN code of the received signal. The frequency \( \omega \)
is the intermediate frequency carrier to which the L1 carrier
signal gets mixed in the RF front end. The term \( n(t) \) is the
noise, its power is \( \sigma^2 \). Normally it can be modeled as
Gaussian, band-limited white noise. \( A^2/2\sigma^2 = SNR \) (where
SNR is signal to noise rate).

The signals of from both channel I and channel Q are
respectively correlated with the local C/A code, and then
multiplied by \( 2\cos(\omega T) \) and \( 2\sin(\omega T) \). The correlation
duration is equal to the cycle of a C/A code. As a result,
channel I gets

\[ I_k = S_i + \text{noise}_i \]

(4)

\[ S_i = \sum_{j=0}^{2^{15}-1} AD(jT)(JT)C_{\text{data}}(jT) \cos(\omega T)/2 \cos(\omega T) \]

(5)

Suppose \( D(t) = 1 \) when these codes are aligned and
correlated. And then gets

\[ S_i = A \frac{\sin \left( \frac{\Delta w N T}{2} \right)}{\sin \left( \frac{\Delta w T}{2} \right)} \cos \left( \frac{\Delta w (N-1) T}{2} + \Delta w TkN \right) + \text{noise}_i \]

(6)

Where N is the sampling number in a C/A cycle, T is the
sampling interval.

Similarly, the summed result of channel Q is:
\[ S_i = A \frac{\sin \left( \frac{\Delta NT}{2} \right)}{\sin \left( \frac{\Delta T}{2} \right)} \sin \left( \frac{\Delta T(N-1)}{2} + \Delta TkN \right) + \text{noise}_{\text{a}}(7) \]

Where \( \text{noise}_{\text{a}} \) and \( \text{noise}_{\text{c}} \) follow a Gaussian distribution \( N(0, 2N\sigma^2) \).

\[ Y = \sum_{i=1}^{M} I_i + Q_i \]

\[ Y = \sum_{i=1}^{M} I_i + Q_i, \]

where \( M \) is the summed times. The use of DF statistics is to avoid the square loss that caused by NCH. Compared with COH, the “differential” in acquisition algorithm based on DF makes it not sensitive to navigation data bit transitions. The “accumulation” after differenced makes the requirement of the accumulation algorithm time shorter, and makes the frequency difference intolerance higher, the acquisition time is not long. So, the acquisition algorithm based on DF has an advantage over COH and NCH on the improvement of SNR.

When codes are aligned, the test statistic \( Y \) is also simplified

\[ Y = \sum_{i=1}^{M} I_i + Q_i = A^2 \frac{\sin^2 \left( \frac{\Delta T}{2} \right)}{\sin^2 \left( \frac{\Delta T}{2} \right)} M \cos(\Delta T N) + \sum_{i=1}^{M} z_i \] (8)

To get the probability density function of \( Y \), the probability density of the random variable \( z_i \) should be got firstly. To do easily the analysis, suppose that \( z_i \) is a Gaussian distribution \( N(0, 4N\sigma^2) \). Hence, \( \sum_{i=1}^{M} z_i \) is a Gaussian distribution \( N(0, 8M N^2 \sigma^2) \). To compensate the errors caused by approximation, the used variance of \( \sum_{i=1}^{M} z_i \), in fact, is equal to \( 8BM N^2 \sigma^4 \).

According to the above analysis, we can determine the acquisition threshold establishing a hypothesis testing.

\[ p = \left( A^2 N^2 M + A^2 CM \right) / 2 \] (9)

Where \( C = \frac{\sin^2(\Delta CT/2)}{\sin^2(\Delta CT/2)} \cos(\Delta CT N) \).

Under the threshold \( p \), we can get wrong acquisition probability \( p_e \). Substituting \( A^2 \) into Equation (9), we get

\[ p = \frac{MN \sigma^2 \text{snr}(N + C) \sqrt{M \beta \text{erfinv}(1 - 2p_e)N^2 + C}}{N^2 - C} \] (10)

Where \( \text{erfinv}(x) \) is the anti-function of

\[ \text{erf}(x) = \frac{2}{\sqrt{n}} \exp(-t^2) \, dt . \]

Equation (10) shows the relation among \( p, M \) and \( p_e \), here \( \beta, N \) and \( C \) is constant. When the wrong acquisition probability is determined, \( p_e \) is also a constant, so \( p \) is only related to \( N \). When the accumulation value exceed the threshold, the code phase and carrier frequency related to this accumulation value is the result we wanted.

**IV. SIMULATION EXPERIMENT**

Figure 1 validates the previous theoretical analysis, and demonstrates that DF suffers from approximately 3dB less processing loss as compared with NCH in the case of low-SNR.

![Figure 1. Processing loss of DF and NCH](image)

The detection probability with data modulation is depicted in Figure 2 for \( M=60, M=80 \) and \( T_{\text{COH}}=1 \text{ ms} \). Again the performance of differentially coherent combining of COH results is about 2dB better than with non-coherent combining of COH results. Under the same processing segment, the number of DF is inherently one less than the number of NCH.

![Figure 2. Probability of detection \( T_{\text{COH}}=1 \text{ ms} \)](image)

Figure 3 and Figure 4 show the effect of increasing the \( T_{\text{COH}} \) (COH time) on two algorithms (DF and NCH). It can be seen that the use of differentially coherent combining of COH results, for each scenario (defined by a specific pair of COH duration and number of DF/NCH accumulations),
unanimously improves the detection probability about 3-4 times around the threshold $C/N_0$ region.

![Figure 3. Probability of detection ($T_{COH}=5$ ms)](image)

![Figure 4. Probability of detection ($T_{COH}=10$ ms)](image)

V. CONCLUSION

We have examined DF processing and evaluated the performance of DF-based detectors. The theoretical analysis illustrates that DF suffers from approximately 3 dB less processing loss as compared with NCH in the case of low post-COH SNR, resulting in the detection sensitivity improvement. The use of DF decision variable can avoid the square loss caused by NCH. The “accumulation” after difference shortens the correlation accumulation time length, and this makes the wrong acquisition rate lower without increasing the data length. In short, the adapted acquisition based on DF has an advantage compared to COH and NCH regarding the improvement of SNR.

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REFERENCES


