I. INTRODUCTION

A major problem in instrumenting high dynamic vehicles with GPS receivers is the difficulty in tracking the GPS signals when the host vehicles are executing severe maneuvers. One way to overcome this problem is to use aiding from an inertial navigation unit on the receiver. Typically, this approach is complex and expensive. Here we present a new receiver concept that enables high dynamic tracking without relying on inertial aiding.

Typical GPS receivers use separate code and carrier phase-locked loops to track time delay, carrier phase, and frequency in order to estimate pseudorange and range rate [1–4]. This approach suffers from the usual loop bandwidth versus dynamic performance tradeoffs commonly encountered in phase-locked loops: the effects of noise increase with increasing loop bandwidth, while dynamic tracking errors increase with decreasing loop bandwidth. For carrier signal-to-noise ratios commonly encountered by GPS receivers on aircraft and missiles, loop bandwidths need to be so narrow that reliable carrier phase tracking can only be achieved for acceleration not exceeding 5 g and jerk (the derivative of acceleration) not exceeding 5 g/s. The receiver described in this paper can operate under much higher dynamics: a breadboard receiver has been built that demonstrated reliable tracking with simulated trajectories corresponding to 150 g acceleration and 150 g/s jerk.

The main new concept described here is the method of estimating and tracking the pseudorange and range rate of a GPS receiver with respect to GPS satellites. This is accomplished by estimating Doppler frequency and code delay in a quasi-open loop, approximately maximum likelihood manner, as opposed to tracking with phase and delay locked loops, as in conventional receivers. This approach enhances the receiver’s ability to track under high receiver dynamics and naturally leads to an all digital implementation.

The approach is based on the premise that over a sufficiently short observation interval the signal parameters may be viewed as constant, though unknown, quantities. With this assumption, maximum likelihood estimation yields the best performance. Therefore, as a first step, we derive the structure of the maximum likelihood estimator (MLE) of delay and Doppler frequency embedded in a typical GPS signal. This MLE is approximated in the receiver. The constant parameter assumption is not valid for a large number of observation times, even though it may be accurate for each “short” observation interval. Thus, some form of tracking must be employed in conjunction with maximum likelihood estimation in order to keep the estimates from eventually drifting outside their allowable range. The method presented in this paper tracks Doppler frequency (but not phase) and pseudorange using a digital fading memory tracking filter.
II. THEORETICAL RESULTS

The GPS satellites transmit pseudorandom signals at two L-band frequencies, denoted L1 and L2. Two different pseudorandom signals are used on each satellite, a P code signal with a chip (clock) rate of 10.23 MHz, and a C/A code with period 1023, clocked at 1.023 MHz. Both codes are biphase modulated at 50 bit/s. The L1 carrier is phase modulated by both signals, with the C/A code lagging the P code by 90°. The L2 carrier is modulated by either the P or the C/A signal, but not both at the same time. The L1 carrier frequency is 154 times the P code chip rate, while the L2 carrier frequency is 120 times the P code chip rate. All frequencies are phase coherent. On L1, the C/A signal has twice the power of the P code signal. The carriers are completely separated. We consider only tracking the L1, P code signal, which can be expressed as

\[ s_T(t) = A_T D(t) p(t) \cos(\omega_0 t + \phi_0) \]

where \( A_T \) is the P code amplitude; \( D(t) \) is the data signal at 50 bit/s, \( D(t) = \pm 1 \); \( p(t) \) is the P code signal; \( \omega_0 \) is the L1 radian carrier frequency, and \( \phi_0 \) is the constant but random carrier phase.

This signal is filtered before transmission to restrict its bandwidth to approximately 37 MHz. The receiver observes a noise-corrupted, attenuated, Doppler-shifted version of the transmitted signal, delayed by the propagation delay between satellite and receiver. The receiver has a different clock than the satellite; hence by observing the signal from one satellite only it measures the sum of the clock offset and the signal delay due to range, denoted pseudorange, but cannot separate the two quantities. Standard techniques [2] using GPS signals from several satellites can be employed to separate range from clock offset, hence eliminating the dependence of the final estimates on clock offset. Here we consider parameter estimation as applied to a single received P code signal. We begin with a brief summary of the underlying maximum likelihood estimation technique.

A. Maximum Likelihood Estimator

The received P code signal can be represented over any given data-synchronous \( T \)-second time interval in complex form as [5]

\[ s(t) = \text{Re} \{ \tilde{s}(t) \sqrt{2} \exp(j \omega_0 t) \} \quad (2.1a) \]

\[ \tilde{s}(t) = A p(t - \tau) \exp(j \omega_d t + \theta) \quad (2.1b) \]

where \( \tilde{s}(t) \) is the "complex envelope" of the received narrowband signal, \( \tau \) is the delay, and \( \omega_d \) is the Doppler radian frequency. The random phase \( \theta = \theta_0 + (\pi/2) \cdot [D(t - \tau) + 1] \) consists of an initial phase offset \( \theta_0 \), plus a time-varying component generated by the data modulation. Since the data modulation does not change during a synchronous time interval, \( \theta \) may be considered an unknown, but constant, parameter. The signal parameters \( (A, \theta, \tau, \omega_d) \) are assumed to change slowly enough to be considered constant over any observational interval. Since the transmitted carrier frequency is known, the complex envelope contains all available information about the signal parameters.

The received signal is observed in the presence of additive, stationary, narrowband Gaussian noise \( n(t) \), with representation

\[ n(t) = \text{Re} \{ \tilde{n}(t) \sqrt{2} \exp(j \omega_0 t) \}. \quad (2.2) \]

Again, \( \tilde{n}(t) \) is the complex envelope of the narrowband noise process. In order to simplify the analysis, we assume that the passband is significantly greater than the signal bandwidth, in which case the noise covariance function may be approximated by

\[ K_n(\xi) = \frac{1}{T} \mathbb{E} \{ \tilde{n}(t + \xi) \tilde{n}(t) \} = N_0 \delta(\xi) \quad (2.3) \]

where \( N_0 \) is the two-sided spectral level of the complex envelope. Given \( \tilde{s}(t) \), the noise-corrupted signal envelope may be represented in the mean-squared sense by an infinite series as

\[ \tilde{r}(t) = \sum_{i=1}^{\infty} \tilde{r}_i \phi_i(t), \quad -\frac{T}{2} \leq t \leq \frac{T}{2} \quad (2.4a) \]

\[ \tilde{r}_i = \int_{-T/2}^{T/2} [\tilde{s}(t) + \tilde{n}(t)] \phi_i^*(t) dt = \tilde{s}_i + \tilde{n}_i \quad (2.4b) \]

where \( \{\phi_i(t)\} \) is a complete set orthonormal on \((-T/2, T/2)\). The limiting form of the covariance function defined in (2.3) can be expressed in terms of this orthonormal basis as

\[ K_n(\xi) = N_0 \sum_{i=1}^{\infty} \phi_i(t + \xi) \phi_i^*(t), \quad -\frac{T}{2} \leq (t, t + \xi) \leq \frac{T}{2} \quad (2.5) \]

(Mercer's theorem). The white noise approximation (equation 2.3) implies that the noise samples are uncorrelated: for complex Gaussian processes it follows that the noise samples are independent. Letting \( \tilde{r}_N(t) \) denote a truncated, \( N \)-term approximation to \( \tilde{r}(t) \),

\[ \tilde{r}_N(t) = \sum_{i=1}^{N} \tilde{r}_i \phi_i(t) \quad (2.6) \]

the joint probability density of the first \( N \) complex samples, conditioned on the signal parameters \( (A, \theta, \tau, \omega_d) \) can be expressed as [5, 6]

\[ p(\tilde{r}_N|A, \theta, \tau, \omega_d) = (\pi N_0)^{-N} \cdot \exp \left( -\sum_{i=1}^{N} |\tilde{r}_i - \tilde{s}_i|^2 / N_0 \right) \quad (2.7) \]

where

\[ \tilde{r}_N = (\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_N). \]
The maximum likelihood estimates of the signal parameters, based on this N-dimensional observation vector, are those values that simultaneously maximize the joint conditional density defined in (2.7). Equivalently, it suffices to maximize the log-likelihood function \( \Lambda(\hat{\mathbf{r}}_N) \), obtained by taking the natural logarithm of the joint conditional density, and discarding terms that contain no information about the parameters of interest. Thus,

\[
\Lambda(\hat{\mathbf{r}}_N) = -\frac{1}{N_0} \sum_{i=1}^{N} |\hat{r}_i - \delta_i|^2.
\]  

(2.8)

The log-likelihood function for the continuous case is obtained from (2.8) by taking the limit as \( N \) grows without bound. Using (2.4b) and (2.5), it follows that

\[
N_0 \Lambda[\hat{r}(t)] = \lim_{N \to \infty} N_0 \Lambda(\hat{\mathbf{r}}_N) \\
= - \int_{-T/2}^{T/2} \hat{r}(t) - \hat{s}(t) |^2 dt.
\]

(2.9)

The unknown phase \( \theta \) and amplitude \( A \) are nuisance parameters that must be estimated simultaneously with the desired parameters. The phase \( \theta \) can be estimated by substituting for \( \hat{s}(t) \) in (2.9) and expanding [6]

\[
N_0 \Lambda[\hat{r}(t)] \\
= 2A \Re \left\{ \exp(-j\theta) \int_{-T/2}^{T/2} \hat{r}(t)p(t-\tau)exp(-j\omega_d\tau)dt \right\} \\
- \int_{-T/2}^{T/2} |\hat{r}(t)|^2 dt - A^2T.
\]

(2.10)

Only the first term on the right side of (2.10) contains \( \theta \) explicitly. For any complex \( z \), \( \Re\{e^{-j\theta}z\} \) is maximum with respect to \( \theta \) when \( \theta = \arg(z) \). The maximum likelihood estimate of \( \theta \) is therefore

\[
\hat{\theta} = \arg \left\{ \int_{-T/2}^{T/2} \hat{r}(t)p(t-\tau)exp(-j\omega_d\tau)dt \right\}.
\]

(2.11)

Substituting into (2.10) yields

\[
\max_{\theta} N_0 \Lambda[\hat{r}(t)] \\
= 2A \int_{-T/2}^{T/2} \hat{r}(t)p(t-\tau)exp(-j\omega_d\tau)dt \\
- \int_{-T/2}^{T/2} |\hat{r}(t)|^2 dt - A^2T.
\]

(2.12)

Differentiating with respect to \( A \) and equating to zero yields

\[
\hat{A} = \frac{1}{T} \left| \int_{-T/2}^{T/2} \hat{r}(t)p(t-\tau)exp(-j\omega_d\tau)dt \right|.
\]

(2.13)

Using this value in (2.12), we obtain

\[
\max_{\theta, \hat{A}} N_0 \Lambda[\hat{r}(t)] \\
= \frac{1}{T} \left| \int_{-T/2}^{T/2} \hat{r}(t)p(t-\tau)exp(-j\omega_d\tau)dt \right|^2.
\]

(2.14)

Since the second term contains no information about the signal, it follows that the maximum likelihood estimates of \( \tau \) and \( \omega_d \) are those values \( \hat{\tau} \) and \( \hat{\omega_d} \) that simultaneously maximize

\[
L(\tau, \omega_d) \triangleq \frac{1}{N_0} \left| \int_{-T/2}^{T/2} \hat{r}(t)p(t-\tau)exp(-j\omega_d\tau)dt \right|^2.
\]

(2.15)

In (2.15), \( N_0 \) is used only in determining the estimator performance. Note that \( L(\tau, \omega_d) \) is a function of two independent variables, \( \tau \) and \( \omega_d \); thus its domain can be viewed as a two-dimensional Cartesian plane with coordinates \( \tau \) and \( \omega_d \). Henceforth we shall refer to this coordinate system as the “(\( \tau, \omega_d \)) plane.”

In a real system, the complex envelope may be obtained by performing quadrature decomposition on the received waveform, indicated symbolically by the operation

\[
\hat{r}(t) = [\sqrt{2} \ r(t)exp(-j\omega_0t)]_{LP}
\]

(2.16)

where LP refers to low-pass filtering of the bracketed function. Once the complex envelope has been extracted from the received signal, the operations indicated by equation (2.15) can be performed to obtain maximum likelihood estimates of the desired signal parameters.

Maximization of \( L(\tau, \omega_d) \) is generally carried out over a finite window in the \((\tau, \omega_d)\) plane. This “uncertainty window” must have dimensions large enough to ensure that with high probability the point corresponding to the true parameters is included. With bandlimited signals and finite observation intervals, \( L(\tau, \omega_d) \) is itself a bandlimited function, hence its “main lobe” occupies a nonzero region around its peak value. Thus, it is only necessary to evaluate (2.15) over a finite grid within the uncertainty window, with grid spacing fine enough to ensure that the main lobe is not missed.

B. Estimator Performance

Receiver performance versus signal to noise ratio (SNR) can be evaluated by using high and low SNR approximations. At low SNR, nonlinear estimation is often accompanied by threshold effects, meaning that below some critical SNR catastrophic deterioration in estimator performance begins to occur. In the problem under consideration, performance degradation is caused by the appearance of large noise spikes, possibly resulting in the selection of a point in the \((\tau, \omega_d)\) plane whose coordinates are not related to the parameters embedded in the received signal. At high SNR, bounding techniques can be applied to obtain accurate performance estimates.

Consider the high-SNR case first. In the absence of noise, if the true parameters are located at coordinates \((0, 0)\) the log-likelihood function reduces to
The dependence of \( \Phi(\tau, \omega_d) \) on \( T \) and \( T_c \) (the chip duration) has been suppressed to simplify notation. The behavior of \( \Phi(\tau, \omega_d) \) can be understood by evaluating the integral along the delay and Doppler directions:

\[
\Phi(\tau, \omega_d) = \sin(\omega_d T/2) \frac{\omega_d T}{\omega_d T/2}
\]

where \( N \) is the period of the PN sequence. Equation (2.18b) is the autocorrelation function of a maximal length linear shift register sequence [7], corresponding to the type of sequence used in the demonstration receiver. For large \( N \), \( \Phi(\tau, 0) \) can often be approximated by its limiting form as \( N \to \infty \).

Since, in the absence of noise, the peak of the log likelihood function occurs at the appropriate coordinates, unbiased estimates are obtained. As the noise level increases, the estimates may begin to deviate from their actual values in a random manner. The estimation variance is bounded by the Cramer-Rao bound, according to which the variance of any unbiased estimate must exceed the diagonal elements of the inverse of the information matrix \( J \), whose \((i,j)\)th component is specified by the expression

\[
J_{i,j} = -E \frac{\partial^2 \Phi(\psi)}{\partial \psi_i \partial \psi_j} \tag{2.19}
\]

where the vector \( \psi \) consists of the desired parameters, and \( L \) is given by (2.15). Recall that in our notation, \( L \) is the log-likelihood function.

It has been shown [3] that the information matrix can be expressed as

\[
J = \begin{bmatrix} \omega^2 & \overline{\omega t} \\ \overline{\omega t} & \overline{t^2} \end{bmatrix} 2A^2T \frac{1}{N_0} \tag{2.20a}
\]

where

\[
\overline{\omega^2} \triangleq \frac{1}{2\pi T} \int_{-\infty}^{\infty} \omega^2 |P(j\omega)|^2 d\omega \tag{2.20b}
\]

\[
\overline{t^2} \triangleq \frac{1}{T} \int_{-T/2}^{T/2} t^2 |p(t)|^2 dt \tag{2.20c}
\]

\[
\overline{\omega t} \triangleq \text{Im} \left[ \frac{1}{T} \int_{-T/2}^{T/2} tp(t) \frac{d}{dt} p^*(t) dt \right] \tag{2.20d}
\]

and

\[
P(j\omega) \triangleq \int_{-T/2}^{T/2} p(t)e^{-j\omega t} dt. \tag{2.20e}
\]

For the problem under consideration, \( \overline{\omega t} = 0, \overline{t^2} = T^2/12 \), and assuming that \( p(t) \) is a time function that has been filtered by a third-order Butterworth filter, \( \overline{\omega^2} = C_\omega(2\pi T_c) \). The details of these calculations are presented in the Appendix, where \( \overline{\omega^2} \) is also evaluated. Substituting into (2.20a), inverting the information matrix and using the diagonal elements, the following bounds are obtained:

\[
\sigma^2_{\tau} \geq \frac{N_0}{2E_s} \frac{1}{\overline{\omega^2}} s^2 \tag{2.21a}
\]

\[
\sigma^2_{\omega_d} \geq \frac{N_0}{2E_s} \frac{1}{\overline{t^2}} (\text{rad/s})^2 \tag{2.21b}
\]

\[
E_s = A^2T. \tag{2.22}
\]

The performance of unbiased maximum likelihood estimators is well approximated by the Cramer-Rao bound at high SNRs. However, at low SNR, performance is often dominated by nonlinear effects that are not adequately described by high-SNR models. The nonlinear effects pertaining to the problem under consideration are discussed next.

The operation of the maximum likelihood estimator under low SNR conditions can be described in terms of "coherence cells" in \((\tau, \omega_d)\) plane. The approximate dimensions of a coherence cell can be established by considering the properties of the two-dimensional noise process

\[
n_z(\tau, \omega_d) = \int_{-T/2}^{T/2} n(t)p(t-\tau)exp(-j\omega_d t) dt. \tag{2.22}
\]

This is a zero-mean process with two-dimensional correlation function

\[
R_{n_z}(\Delta \tau, \Delta \omega_d) \triangleq \text{cov} \{n_z(\tau+\Delta \tau, \omega_d+\Delta \omega_d), n_z(\tau, \omega_d)\} = N_0 T \Phi(\Delta \tau, \Delta \omega_d) \tag{2.23}
\]

where \( \Phi \) is defined in equation (2.17b). The last equality follows from the white-noise approximation. Note that for large \( N \), the first zeros along the \( \omega_d \) and \( \tau \) directions occur roughly at \( \omega_d = 2\pi/T \) rad/s and \( \tau_0 = T_c s \), suggesting the approximation of a coherence cell in the \((\tau, \omega_d)\) plane by a rectangle of dimensions \( \omega_d \sigma_T = \omega_d \tau_0 \) and \( \tau_0 \sigma_T \).

Since \( R_{n_z}(0, 0) = N_0 T \), the noise process over each coherence cell may be treated as a distinct zero-mean complex Gaussian random variable with variance \( \sigma^2 = N_0 T \). We observed before that the signal, when present, also occupies an area corresponding roughly to the area of a coherence cell in the delay-frequency plane. In the absence of noise, its mean value is \( AT \). Thus, a rectangular observation region of dimensions \( \Omega_0 rad/s \) and \( T_0 s \) may be said to contain disjoint coherence cells, each of which can be treated as though it contained a single Gaussian random variable, statistically independent of all others.
Performance can be determined by observing that the same values of $\tau$ and $\omega_0$ maximize both the magnitude square and the magnitude of the complex transform defined in (2.15). The probability density of the magnitude of the sum of a signal component (amplitude $AT$) and complex Gaussian noise (variance $\sigma^2$) is the Rice density

$$p_1(x) = \frac{2x}{\sigma^2} \exp\left(-\frac{A^2T^2 + x^2}{\sigma^2}\right) I_0\left(\frac{2ATx}{\sigma^2}\right), \quad x \geq 0$$

(2.25)

which reduces to the Rayleigh density as the signal amplitude becomes arbitrarily small:

$$p_2(x) = \lim_{A \to 0} p_1(x) = \frac{2x}{\sigma^2} \exp(-x^2/\sigma^2), \quad x \geq 0.$$  

(2.26)

The probability $p$ of selecting the correct coherence cell is simply the probability that the magnitude of the desired random variable (composed of signal plus noise) exceeds the magnitudes of all other random variables in the observation region. Therefore,

$$p = \int_0^\infty \int_0^x p_1(x) \int_0^x p_2(y) dy \, dx$$

(2.27)

while the probability of selecting a coherence cell containing noise alone is just $q = 1 - p$. Since

$$\int_0^x p_2(y) dy = 1 - \exp(-x^2/\sigma^2)$$

(2.28)

it follows that

$$q = 1 - \int_0^\infty \frac{2x}{\sigma^2} \exp\left(-\frac{A^2T^2 + x^2}{\sigma^2}\right) I_0\left(\frac{2ATx}{\sigma^2}\right) [1 - \exp(-x^2/\sigma^2)]^{M-1}.$$  

(2.29)

The probability of selecting the wrong coherence cell $q$ depends only on the signal to noise ratio ($A^2T^2/\sigma^2$), and on the total number of relevant coherence-cells $M$. The behavior of $q$ as a function of SNR is depicted in Fig. 1 for various values of $M$. In the demonstration receiver, ($\Omega_0/\omega_0$) = 32 and ($T_0/\tau_0$) = 5; hence $M = 160$. Since $T = 20$ ms, SNR is defined here as $A^2/N_0$ in dB-Hz. Note that $q$ is identical to the symbol error probability that occurs in the incoherent detection of $M$-ary orthogonal symbols corrupted by additive Gaussian noise.

If an error is committed in locating the signal, then the variance of the estimation error can be calculated by observing that each point within the observation region has an equal a priori probability of being selected by the maximum likelihood estimator. Therefore, upon erring,

$$\sigma_{q\text{Rice error}}^2 \frac{T_0}{12} = \frac{T_0^2}{12} s^2$$

(2.30a)

At any SNR, the variance of the estimation error can be expressed as

$$\begin{align}
\sigma_e^2 &= q(\sigma_{q\text{cell error}}^2) + p(\sigma_{p\text{cell error}}^2) \\
\sigma_e^2 &\geq \frac{T_0^2}{12} + p \frac{N_0}{2E_s} \frac{1}{\omega^2} s^2 \\
\sigma_e^2 &\geq \frac{T_0^2}{12} + q \frac{N_0}{2E_s} \frac{1}{\omega^2} (\text{rad/s})^2.
\end{align}$$

(2.31a, 2.30b)

The behavior of these bounds as functions of SNR are shown in Fig. 2(a) and (b), in units of m and m/s to facilitate comparison with experimental and simulation results. Unit conversions are based on the observation that $10^{-7}$ s (roughly one chip time) corresponds to 29.8 m, while one FFT filter bandwidth (50 Hz) corresponds to 9.5 m/s in range rate. The dimensions of the uncertainty region were chosen to be $T_0 = 29.8 \times 5 = 149$ m and $\Omega_0 = 9.5 \times 32 = 304$ m/s, in agreement with those of the demonstration receiver. The actual values of the rms pseudorange and range rate errors can be determined at SNRs for which either $p$ or $q$ is negligible. For example, at 20 dB-Hz $p$ is small and the first terms in (2.31b) and (2.31c) dominate. Since $q = 0.9$,

$$\sigma_e \approx \sqrt{qT_0^2/12} \approx 41 \text{ m}$$

and

$$\sigma_{q\text{Rice error}} \approx \sqrt{q(T_0^2/12)} \approx 83 \text{ m/s}.$$  

Similarly, at 40 dB-Hz, $p \approx 1$, hence using the values $T^2 = 3.3 \times 10^{-5}$ s$^2$ and $\omega^2 = (2/\pi) (0.7 \times 10^{14}$ s$^{-2}$ from the Appendix, and converting units we find that

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig. 1. Outlier probability as a function of SNR.}
\caption{Outlier probability as a function of SNR.}
\end{figure}
The new estimates can be used to predict the values of the current input parameters prior to subtraction.

Parameter tracking and prediction is accomplished by means of a digital filter. The filter uses estimates of pseudorange and range rate to compute a state vector consisting of pseudorange, range rate, and acceleration. The filter model is

\[
x_{n+1} = Gx_n + v_n \quad (2.32a)
\]

\[
y_n = Hx_n + u_n \quad (2.32b)
\]

where \(x\) and \(y\) are state and measurement vectors, and \(G\) and \(H\) are state transition and measurement matrices, respectively. The noise vectors \(v\) and \(u\) are modeled as white Gaussian processes. The statistics of the process noise \(v\) is especially difficult to model since it is derived from vehicle dynamics hence tends to vary significantly for different host vehicles.

Several types of filters were examined for the current application, including a second-order Kalman filter [8], and second- and third-order fading memory filters [9]. A third-order fading memory filter was selected due to its superior response to vehicle dynamics, good noise suppression, and ease of implementation. The fading memory filter is a least squares estimator that applies an exponentially decaying weight to past measurements. Typically, it is less sensitive to mismodeling problems and tends to be more stable than typical Kalman filters. The selected state model is

\[
x = \begin{bmatrix} \text{pseudorange} \\ \text{range rate} \\ \text{acceleration} \end{bmatrix}
\]

\[
y = \begin{bmatrix} \text{pseudorange measurement} \\ \text{range rate measurement} \end{bmatrix}
\]

\[
G = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\]

where \(T\) is the signal integration time. State estimate \(\hat{x}\) is computed using

\[
M_{n+1} = H^TWH + e^{-T^*G^{-1}}M_nG^{-1} \quad (2.34a)
\]

\[
W = \begin{bmatrix} 1 & 0 \\ 0 & w^* \end{bmatrix}
\]

The bounds of (2.31) are believed to be tight at all SNRs, because estimation performance approaches the Cramer-Rao bound at high SNR, while below threshold (or at "low-SNR") the probability of selecting the correct coherence cell rapidly becomes negligibly small. The above results are predicated on the assumptions that the parameters of interest are constant, and located near the center of the uncertainty window.

C. Tracking Loop Description

In a dynamic environment, the uncertainty window of the maximum likelihood estimator must be continuously centered in order to keep the estimates within the allowable limits. The rate at which this centering operation is carried out depends on the window dimensions as well as on the particular vehicle trajectories involved. The implementation is by means of a tracking loop as shown in Fig. 3. This loop effectively subtracts out predictions of the parameters from the signal, thus keeping the values of the current estimates near zero (that is, near the center of the uncertainty window). If the required computation introduces a significant delay, then
\[ \hat{x}_{n+1} = G\hat{x}_n + M_n^{-1}H^T W(y_n + 1 - HG\hat{x}_n) \]  

(2.34c)

where \( \tau^* \) is the filter time constant and \( w^* \) is the ratio of the weight of the range rate measurement to that of the pseudorange measurement. At each step, a new value of \( M_n \) is computed, followed by the derivation of a new state estimate. Filter implementation, selection of parameters, and performance were examined in [10].

III. IMPLEMENTATION AND SIMULATION

In this section we describe the demonstration receiver, the instrumentation built to test the receiver, and the simulation program developed to evaluate receiver performance.

A. Receiver Description

The receiver consists of a radio frequency (RF) front end, baseband digitizers, and signal processing equipment. The RF front end mixes the input signal to in-phase and quadrature baseband using a reference signal at the nominal L-band frequency. The in-phase and quadrature signal components are filtered by third-order Butterworth low-pass filters with 10 MHz passbands. The complex baseband signal is then quantized by two 3-bit A/D converters. The signal processing equipment, described in detail in the following paragraphs, uses a combination of custom hardware and a commercial microcomputer.

The implemented version of the maximum likelihood estimator is termed AMLE (approximate maximum likelihood estimator) in order to distinguish it from the continuous estimator structure defined in Section II. The basic idea of the AMLE is to approximate the operation of the two-dimensional maximum likelihood estimator using a discrete correlator for delay and a fast Fourier transform (FFT) for frequency estimation. A main consequence of using the AMLE is that the estimator works only when the signal delay and frequency fall within the delay and frequency window determined by the computed correlation lags and FFT frequencies. A block diagram of the demodulation and signal processing functions is shown in Fig. 3. The main functional blocks are the predict removal block, the AMLE, the predict addition block, the tracking filter, and the predictor. All variables in the figure are column vectors composed of pseudorange, range rate, and acceleration.

The predict removal block, implemented in hardware and software, centers the AMLE window at the predicted pseudorange and range rate values. Use of predict removal means that the AMLE window does not have to cover the full extent of possible pseudorange and Doppler frequencies. Instead, the window needs to cover only the maximum range of the prediction errors. The predict removal block performs three specific functions: range rate predict removal, pseudorange predict removal, and pseudorange stepping each correlation time to compensate for possible changes during a bit time. Range rate predict removal is implemented with a digital complex multiplier, which multiplies the complex I-Q baseband samples by a sinusoid of the predicted frequency. Pseudorange predict removal and stepping is implemented by controlling the delay of the code generator driving the correlator.

The AMLE processing consists of three steps, performed each data bit time. First, every 1/1600 s, or 32 times per bit, correlation values are computed for eleven code lags, separated by a half P code chip. Second, a FFT is computed for each lag and the peak power over the AMLE window is identified. Third, interpolation formulas are applied to estimate the location of the peak relative to the discrete points where the likelihood function is evaluated.

Correlation is performed by multiplying the noisy complex samples by an unfiltered, shifted version of the code sequence, and summing. For each lag, 32 complex zeroes are appended to the 32 complex correlation values over the data bit time, and a 64-point complex FFT is performed. The effective FFT filter bandwidths are 50 Hz, spaced 25 Hz apart. This results in a 64 by 11 array of FFT values. The matrix spans 1575 Hz (63 multiplied by 25 Hz) in frequency, and 149 m (14.9 m multiplied by 10) in pseudorange. The maximum of the discrete AMLE function is defined as the number with the greatest magnitude, excluding the first and last lags. Once the maximum is identified, finer pseudorange and range rate estimates are obtained by interpolation.

The interpolation formula for pseudorange estimation uses the location and value of the peak AMLE amplitude and its two closest neighbors in the correlation domain to define an isosceles triangle. The algorithm uses the two adjacent points defining the greatest slope to define one side of the isosceles triangle, and a line with the negative slope through the third point to define the other equal side. The location of the peak is taken to be the refined estimate of delay, which is easily related to pseudorange. The interpolation formula for range rate uses the closest neighbors of the AMLE peak in the frequency domain to define a parabola, and uses the location of its peak to estimate Doppler frequency, hence range rate.

The output of the AMLE is the difference between the actual and predicted pseudorange and frequency. The predict addition block, implemented in software, adds the predicted pseudorange and predicted frequency to the AMLE output, resulting in estimates of the input pseudorange and frequency.

The AMLE provides raw estimates of pseudorange and range rate once every data bit time, or 20 ms. A tracking filter is used to achieve three objectives: first, to derive the system state from the observables (e.g., estimate platform acceleration), second, to smooth the random noise effects, and third, to predict pseudorange and range rate for the purpose of positioning the AMLE window. The tracking filter accepts pseudorange and range rate data, and calculates filtered estimates of pseudorange, range rate, and acceleration. Filter selection
depends on tradeoffs between noise suppression and the filter's ability to track high platform dynamics without losing lock. Dynamic response and noise suppression properties of the various tracking filters examined for possible use in the demonstration system were evaluated in [10]. The filter selected for the demonstration is a third-order fading memory filter with parameters \( \tau^* = 0.14 \text{ s} \) and \( \omega^* = 0.1 \text{ s}^{-2} \).

The predictor generates pseudorange and range rate predictions used to close the feedback loop. The prediction consists of two steps: compensation for the loop delay, and quantization of the prediction to discrete values. As implemented, the forward data path has a delay of three data bit times, i.e., 60 ms, caused by the pipelining of the various processing functions. Unquantized predictions, corrected for this delay, are given by

\[
x_p = G^3 x_f
\]

where \( x_f \) is the state estimate, \( x_p \) the predicted value, and \( G \) is defined in (2.33b). The pseudorange predict is quantized to the nearest pseudorange lag, i.e., to the nearest multiple of 14.9 m, while the range rate predict is quantized to the nearest multiple of a FFT filter bandwidth, corresponding to 9.5 m/s in range rate.

B. Test Instrumentation

A test instrumentation subsystem was built to generate test signals and noise for the purpose of demonstrating and evaluating the GPS receiver. A programmable frequency synthesizer was used to generate a clock signal at twice the P code frequency. This frequency was halved to generate the P code clock, and multiplied by 77 to generate a phase coherent L1 carrier signal. A pseudonoise sequence with period 1023 was used to simulate the P code. This code was modulated by 50 bit/s data, and the resultant waveform used to biphasemodulate the L1 carrier. The signal was bandpass filtered (with a bandwidth of 37 MHz) to simulate the satellite transmitter, and broadband noise was added to simulate receiver noise, thus achieving any desired SNR. The estimated accuracy in setting SNR is \( \pm 0.5 \) to \( \pm 1 \) dB. Different receiver dynamics were simulated by programming the frequency synthesizer, resulting in properly phase coherent carrier and code signals.

C. Simulation Description

A software simulation of the receiver was conducted to independently confirm theoretical and experimental performance. In the simulation, the relevant operations performed by the actual demonstration system were modeled, taking into account the various degradations and losses suffered by the signal as it propagates through the system. Once a reliable correspondence has been established between simulation predictions and hardware results, additional simulations could be used to verify potential improvements offered by extensions and modifications to the current system, before incorporating the proposed changes into the hardware.

The simulation program reads values of acceleration, velocity, and range from a data file, and from these computes the corresponding values of frequency and delay. Using tabulated values of the filtered signal cross-correlation function and an array of simulated correlated noise samples, the program generates eleven numerical sequences, each representing a cross-correlation of the received filtered noisy signal with an unfiltered, noiseless signal sequence, delayed by an integer number of lags. A total range of \( \pm 5 \) lags is covered. Next, a complex FFT is performed on each sequence, its magnitude obtained, and the largest element determined. The coordinates of the largest element, properly scaled, represent rough estimates of incremental pseudorange and range rate. Using the values of the neighboring points, the numerical interpolation algorithms of the demonstration receiver are used to obtain refined incremental estimates. Quantized predicts from the previous estimates are added, and the resulting parameter estimates entered into the tracking filter. Finally, updated quantized predicts are obtained from the filtered estimates, delayed, and subtracted from the input trajectories, thus closing the loop on the tracking operation.

The output of the simulation is a time history of pseudorange and range rate estimation errors for each simulated trajectory. Enough samples are obtained to allow the calculation of sample statistics over various parts of the trajectory. In the examples to follow, 1000 consecutive samples were used for computing simulated error statistics.

IV. RESULTS

The demonstration system and the software simulation described in Section III were used to validate the theoretical results of Section II. Each test was characterized by an SNR and a simulated trajectory. Two types of simulated trajectories were used. Time history plots for the first type, step acceleration, are shown in Fig. 4. The range-rate plots have 6 s duration positive and negative ramps, separated by 8 s of constant range rate. The transitions from constant range rate to ramp range rate correspond to steps in acceleration that, although unlikely to occur in practical systems, can be viewed as a limiting case of high dynamic events. The second type of trajectory, circular motion, is shown in Fig. 5. In this case, circular motion is projected onto a straight line, with the result that pseudorange, range rate, and acceleration all become sinusoidal waveforms. A typical circular motion trajectory has a period of 8 s, hence the peak jerk (derivative of acceleration) is \( 2\pi/8 \) times the peak acceleration, in consistent units. The trajectory of Fig. 5 has peak acceleration of 50 g and
signal strength, the input SNR with a 0 dBi antenna is 40 dB-Hz. Since antennas on aircraft and missiles can have very low gain in some directions, it is important for the receiver to operate well below 40 dB-Hz, say as low as 30 dB-Hz. On the other hand, a combination of very high antenna gain and strong satellite signal might result in SNRs as high as 50–60 dB-Hz. The receiver is not required to achieve the full theoretical benefit of very high SNRs in these regions, but need only perform as well as at nominal (40 dB-Hz) signal strengths. Thus, we are primarily concerned with SNRs in the region 30–40 dB-Hz, provided that no anomalous behavior occurs at much higher SNRs.

B. Reconciliation of Losses

The dominant loss factors are explained in the following paragraphs, and summarized in Table I. All of the losses need to be applied in reconciling the theoretical and experimental results. These losses total 1.5 dB ± 1.0 dB. The simulation accounts for all losses except those indicated by an asterisk in Table I, i.e., the transmitter passband and the quantization effects. Losses not accounted for in the simulation total 0.4 dB ± 0.3 dB.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter filter (*) (gain due to definition of carrier power)</td>
<td>-0.3 ± 0.1</td>
</tr>
<tr>
<td>Correlation loss due to bandwidth in receiver</td>
<td>0.5 ± 0.1</td>
</tr>
<tr>
<td>Analog to digital converter, 3 bit (*)</td>
<td>0.3 ± 0.1</td>
</tr>
<tr>
<td>Reference sine wave, 3-level quantization (*)</td>
<td>0.4 ± 0.1</td>
</tr>
<tr>
<td>FFT filter loss, offset from center frequency</td>
<td>0.3 ± 0.1</td>
</tr>
<tr>
<td>Dynamic losses (50 g acceleration, 50 g/s jerk)</td>
<td>0.3 ± 0.2</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>0.0 ± 0.3</td>
</tr>
<tr>
<td>Total SNR losses</td>
<td>1.5 ± 1.0</td>
</tr>
</tbody>
</table>

The theoretical analyses assume that the P code signal is not bandlimited. The actual transmitted P code signal is bandlimited to 37 MHz, or ± 18.5 MHz about the carrier frequency. Thus, the transmitter filter reduces the signal power by 0.3 dB. Since the specified power is the power in the 37 MHz band, the signal power density within the band is 0.3 dB higher than if the total power were not bandlimited. Since the power near the carrier frequency dominates the solution, there is an effective gain of 0.3 dB ± 0.1 dB.

After being mixed to baseband, the signal is low pass filtered prior to A/D conversion. This reduces the peak of the cross-correlation function between the input signal and the local P code by 0.8 dB. The noise is also affected, since the correlator noise is the product of the local P code and the filtered receiver noise. The spectral density of this product noise, at zero frequency, is 0.3 dB less than \( N_0 \), the receiver noise density at zero frequency.

A. Input Signal to Noise Ratio

The signal to noise ratio of interest for performance analysis is the ratio of code power to noise spectral density at the output of the receiver front end, i.e., at the input to the signal processing equipment. This SNR is determined by the satellite signal strength at the input to the antenna, the antenna gain at the proper polarization, and the system noise figure. The GPS signal strength is specified such that the power at the output of a 0 dBi antenna with right circular polarization (RCP) is at least −133 dBm for L1, P code. This is met whenever the satellite elevation is over 5° and the atmospheric loss is less than 2 dB.

A total system noise figure of 3.5 dB is practical. This corresponds to a system temperature of 359 K. For a 3.5 dB noise figure and minimum specified L1, P code

peak jerk of 39.27 g/s (rounded to 40 g/s). The results of these tests are presented below.

Fig. 4. Measured response to 50 g step acceleration, SNR = 34 dB-Hz.

Fig. 5. Measured response to 50 g, 40 g/s circular motion, SNR = 33 dB-Hz.
The net effect is a reduction in effective SNR by \(0.5 \pm 0.1\) dB.

The next loss is due to signal quantization. A three-bit analog to digital converter is used. With optimum signal level, a 3-bit A/D converter would have a loss of 0.2 dB. We conservatively estimate the actual loss to be 0.3 dB \(\pm 0.1\) dB, averaged over the likely range of input signal levels. There is also loss due to a 3-level quantization of the reference sine waves at the input to the digital complex multiplier. This loss is 0.4 dB, since the total power in the three-level signal exceeds the power in the fundamental frequency component by this amount. The tolerance is \(\pm 0.1\) dB.

Loss in SNR occurs when the frequency of the signal at the input to the FFT is not at the center frequency of one of the FFT filters. The mechanism uses filters spaced by one-half of the filter bandwidth, with responses crossing over at the \(-1\) dB points. The maximum loss is 1 dB, and the average loss is 0.3 dB \(\pm 0.1\) dB.

There are also losses in SNR due to receiver dynamics. Acceleration causes the signal bandwidth to expand, because the carrier after code removal is effectively a chirped sinusoid. Jerk causes the reference frequency fed back to the AMLE complex multiplier to be in error, which means that the frequency at the correlator outputs is offset from zero. This causes signal attenuation because the \(1/1600\) s correlation time corresponds to a low pass filter. Each of these effects is approximately 0.3 dB maximum and 0.1 dB average with circular motion of 50 g maximum acceleration and 50 g/s maximum jerk. Since jerk is the derivative of acceleration, maximum acceleration occurs when the jerk is zero, and the two losses are not both high at the same time. We conservatively assume an average total loss of 0.3 dB \(\pm 0.1\) dB. (The loss is approximately 1 dB at maximum acceleration and jerk of 100 g and 100 g/s.)

Finally, there may be miscellaneous losses which have not been accounted for, and there may be errors in estimating the above losses. As a result of the simulations and demonstration tests, we believe that an additional tolerance of \(\pm 0.3\) dB is reasonable.

C. Comparison of Results

All of the results presented in this section have been adjusted to account for the losses described above. That is, the theoretical results were shifted by 1.5 dB, and the simulation results by 0.4 dB.

Estimates were obtained at the output of the tracking filter for both the demonstration receiver and the software simulation. For the demonstration receiver, examples of the time histories of pseudorange and range rate errors for the case of 50 g step acceleration and 50 g, 40 g/s circular motion are provided in Figs. 4 and 5, at SNRs of 34 dB-Hz and 33 dB-Hz. At these SNRs, outliers virtually never occur. The error spikes in the range-rate estimate in Fig. 4, and the relatively large sinusoidal range rate errors in Fig. 5 are determined by the response of the tracking filter to the dynamics.

A typical simulation output for 50 g, 8 s circular motion is shown in Fig. 6, at an SNR of 30 dB-Hz. At this lower SNR, outliers frequently occur. Indeed, the randomly occurring spikes apparent in the estimation errors are caused by outliers, which contribute significantly to the total rms error.

In Section II we considered only the AMLE, and the effects of the tracking filter were not taken into account.

Fig. 6. Simulation response to 50 g, 40 g/s circular motion, SNR = 30 dB-Hz.
Therefore, the theoretical results must be effectively propagated through the tracking filter in order to take the filter’s noise reduction properties into account. The pseudorange and range rate noise reduction factors for the third-order fading memory filter employed depend on the ratio of rms range rate to rms pseudorange errors, as shown in Fig. 7. This ratio varies with SNR at low SNRs, when outliers frequently occur. The appropriate noise reduction factor was used at each SNR.

![Fig. 7. Noise reduction factors for the third order fading memory filter.](image)

The theoretical, simulation, and experimental results for pseudorange and range rate at the output of the tracking filter are compared in Figs. 8 and 9. Here the theoretical results correspond to errors in the filtered estimates. These estimates are obtained from Fig. 2 by first applying the noise reduction factors of Fig. 7, followed by a 1.5 dB shift to the right to account for unmodeled system losses. To determine the noise reduction factors, we observe that at 40 dB-Hz, the ratio of rms range rate error to rms pseudorange error is 0.36 s⁻¹. From Fig. 7 this yields pseudorange and range rate noise reduction factors of 3.4 and 1.4, respectively. Thus, using the previous example, the theoretical bounds on rms filtered pseudorange and range rate errors are 0.72/3.4 = 0.21 m and 0.26/1.4 = 0.19 m/s, respectively, at a SNR of 41.5 dB-Hz.

In Figs. 8 and 9, theory, simulation and experiment are all in good agreement at SNRs below 40 dB-Hz. Higher SNRs are also shown to demonstrate the dynamic range of the receiver, in case a favorable combination of high antenna gain and high satellite power should occur. Accuracy at high SNRs is dominated by instrumentation and software implementation losses and by inaccuracies in the interpolation formulas. These instrumentation losses are on the order of 10 cm for pseudorange and 10 cm/s for range rate.

Experimental results were also obtained for circular motion covering a wide range of dynamic conditions (from 3 g, 2.4 g/s to 150 g, 157 g/s). These results are displayed in Fig. 10. The theoretical results for Fig. 8 are included for comparison. The results were also confirmed by simulation. It is apparent that high dynamics do not degrade the pseudorange estimator performance significantly for SNRs below 40 dB-Hz, the range of greatest practical interest. For 100 g acceleration and SNR above 40 dB-Hz, minimum instrumentation errors approach 20 cm, compared with roughly 10 cm for 50 g acceleration. The results for range rate estimation errors were similar. Thus, we conclude that the receiver functions over a wide range of dynamic conditions without suffering significant deterioration in performance. In fact, good operation was demonstrated for circular trajectories corresponding to 150 g acceleration and 157 g/s jerk.

![Fig. 8. Pseudorange error versus SNR for 50 g, 40 g/s circular motion.](image)

![Fig. 9. Range rate error versus SNR for 50 g, 40 g/s circular motion.](image)

![Fig. 10. Pseudorange error versus SNR for circular motion.](image)
V. CONCLUSIONS

This work presents a new GPS receiver concept for tracking delay and Doppler frequency when the receiver is under high dynamics. It shows conclusively that pseudorange and range rate tracking is possible without inertial aiding at accelerations up to 150 g and jerk up to 157 g/s. It also shows that tracking at 50 g and 40 g/s is possible at input SNRs as low as 28 dB-Hz, which is 12 dB below the SNR achieved with minimum satellite signal strength, 0 dBi antenna gain, and 3.5 dB system noise figure.

APPENDIX

The expressions for \( \overline{\omega}^2 \) and \( \overline{r}^2 \) are derived. First consider \( \overline{\omega}^2 \). Assuming that \( p(t) \) is a filtered PN waveform, its power spectrum \( S_p(\omega) \) can be expressed as

\[
S_p(\omega) = S_{p_0}(\omega) |H(j\omega)|^2
\]

where \( S_{p_0}(\omega) \) is the power spectrum of an unfiltered PN waveform consisting of positive and negative going square pulses of unit amplitude, and \( H(j\omega) \) is the transfer function of the filter. For a third-order Butterworth filter

\[
|H(j\omega)|^2 = \frac{1}{1 + (\omega W_0)^6}.
\]

If \( p_0(t) \) is a maximal length linear shift register sequence of period \( N \) and chip duration \( T_c \), then its power spectrum is the Fourier transform of the autocorrelation function defined in (2.18b). For large \( N \), we can use the power spectrum of the limiting form of the autocorrelation function.

\[
S_{p_0}(\omega) = \frac{1}{T_c} \frac{\sin^2(\omega T_c/2)}{(\omega T_c/2)}.
\]

According to the classical definition of the power spectrum for a deterministic function, we can also write

\[
S_p(\omega) = \lim_{T \to \infty} \frac{1}{T} |P(j\omega)|^2
\]

where

\[
P(j\omega) = \int_{-T/2}^{T/2} p(t) e^{-j\omega t} dt
\]

as in (2.20e). For \( T_c \ll T \), we obtain the approximate expression

\[
|P(j\omega)|^2 \approx T S_p(\omega).
\]

Substituting for \( |P(j\omega)|^2 \) in (2.20b), and using (A1) and (A3) yields

\[
\overline{\omega}^2 = \frac{T_c}{2\pi T} \int_{-\infty}^{\infty} \frac{T \omega^2 \sin^2(\omega T_c/2)}{(\omega^2 T_c^2/4) \left[ 1 + (\omega/W_0)^6 \right]} d\omega
\]

where \( C_\omega \) is the value of the integral. With \( T_c = 10^{-7} \) s and \( W_0 = 2 \pi \times 10^7 \) rad/s, numerical integration yields \( C_\omega = 6.7 \times 10^7 \), hence \( \overline{\omega}^2 = (2/\pi)6.7 \times 10^14 = 4.3 \times 10^{14} \).

Next consider \( \overline{r}^2 \). For an ideal PN sequence \( |p_0(t)| = 1. \) In our case this equality is not strictly true, but if the filter bandwidth is large enough to pass the significant spectral components of \( p(t) \), then we can use the approximation \( |p(t)| = 1 \), and obtain

\[
\overline{r}^2 = \frac{1}{T} \int_{-T/2}^{T/2} |p(t)|^2 dt
\]

\[
\approx \frac{1}{T} \int_{-T/2}^{T/2} t^2 dt = \frac{T^2}{12}.
\]

With \( T = 0.02 \) s, \( \overline{r}^2 = 3.3 \times 10^{-5} \) s. These are the values used in equations (2.21) for obtaining the theoretical results.

REFERENCES

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