I. INTRODUCTION

GPS (Global Positioning System) carrier-phase measurements are subject to a range of error sources and noise. Most of the systematic biases such as tropospheric and ionospheric delay can be removed by differential GPS (DGPS) techniques. Multipath error on the carrier-phase measurement, however, does not correlate between antenna locations. Carrier multipath is very specific to the antenna phase center location with respect to the surrounding environment and the satellite-receiver antenna phase center geometry. As a result, carrier-phase multipath is a major impediment to high accuracy differential techniques [2].

GPS multipath is modeled using equations that characterize it as a function of amplitude, delay, and phase with respect to the direct signal. The purpose of this paper is to compare the theoretical carrier-phase multipath error envelopes with bench test data for the cases of coherent and noncoherent code tracking loops based on the standard (1 chip) and narrow (0.1 chip) correlator architectures. In addition, the performance of a GPS receiver running a coherent code-tracking loop under conditions of multipath fading is also presented. This will serve to highlight critical aspects of the tracking loop design.

Sections II and III provide a review of the theoretical characterization of carrier-phase multipath. Section IV describes the bench test setup used to collect GPS carrier-phase data. Section V compares the bench test data with the theoretical characterization provided in Sections II and III. Included in this section are analyses of the bench test data and the results of this validation effort. Section VI deals with high fidelity modeling and comparison of this model with the bench test results. Section VII includes an analysis of multipath relative phase rate and its effects on the code and carrier tracking loops. A carrier-aided code-tracking loop is implemented to mitigate the effect of fading multipath on pseudo-range code tracking accuracy. Conclusions are provided in Section VIII.

II. CHARACTERIZATION OF PHASE MULTIPATH

In the absence of multipath, the receiver is able to track the direct incoming signal phase. However, in the presence of multipath, the tracking loops in the GPS receiver follow the composite incoming signal (direct plus multipath). In addition, the input to the correlation process is the composite signal rather than the desired direct component. The receiver’s tracking loops are unable to differentiate between the direct and composite signals. They employ the concept of null tracking which as a result yields a non-zero error in the estimate of the direct signal time-of-arrival and carrier phase.
Various methods have been used to characterize multipath. The interaction of the direct and reflected signals (specular reflections) can be analyzed in terms of correlation functions. The ideal correlation function is scaled in amplitude and shifted in time to represent the multipath component of the signal. Care is taken to ensure that the phase of the multipath signal (with respect to the direct) is also preserved in this process. It should be noted that, for the analysis in this paper, the correlation function sidelobes are assumed to be zero. The impact of non-zero sidelobes has been described in [3].

Carrier-phase multipath can be studied by analyzing a simple phasor diagram (Fig. 1) [4]. Without loss of generality, the direct signal is assumed to have zero-phase. The phase of the multipath with respect to the direct is given by the angle $\theta_m$. This representation allows one to resolve the multipath phasor into its in-phase (along the direction of the direct signal) and quadrature components.

The magnitudes of the direct and multipath phasors are given by

$$D = R(\tau_e)$$

$$M = \alpha R(\tau_e - \delta)$$

where $R(\tau_e)$ is the correlation function for the pseudorandom noise (PRN) code at time lag $\tau_e$, $\delta$ is the delay of the multipath with respect to the direct, and $\alpha$ is the ratio of multipath-to-direct signal (otherwise referred to as the multipath-to-direct ratio: M/D). $\tau_e$ is the code tracking error of the DLL (delay locked loop). In order to determine the mathematical model for the composite phase $\theta_c$, the multipath is decomposed into its in-phase component, $M_I = M \cos(\theta_m)$, and quadrature component, $M_Q = M \sin(\theta_m)$. The correlation values used above are from the prompt correlator.

Having resolved the multipath into its components (Fig. 2), it is now possible to determine the phase relation between the composite vector (direct plus multipath) and the direct signal. This quantity (represented in radians or fractions of a wavelength) is the error due to carrier-phase multipath, assuming that the receiver’s carrier tracking loop is perfectly locked onto $\theta_c$. Utilizing the arctan relation in order to obtain the composite phase (similar to the manner in which the phase of a complex quantity is estimated), the carrier-phase multipath error is given by

$$\theta_c = \arctan \left( \frac{M_Q}{D + M_I} \right).$$

Rewriting (3) in terms of the correlation function by substituting (1) and (2) [5],

$$\theta_c = \arctan \left( \frac{\alpha R(\tau_e - \delta) \sin(\theta_m)}{R(\tau_e) + \alpha R(\tau_e - \delta) \cos(\theta_m)} \right)$$

where

- $\theta_m$ = phase of the multipath with respect to the direct,
- $\theta_c$ = composite phase tracking error,
- $\tau_e$ = code tracking error in chips,
- $\delta$ = delay of the multipath with respect to the direct.

For M/D $\leq$ 0 dB, the maximum possible phase-tracking error is $\pi/2$ rad or a quarter wavelength. At the GPS L1 frequency this is approximately 4.8 cm [5]. From (4), it can be seen that the magnitude of this carrier-phase multipath error (in radians) is independent of the choice of frequency (L1/L2). It is a function of M/D ratio and phase of the multipath with respect to the direct.

This expression for the phase error as derived earlier is a complete characterization of carrier-phase multipath. The following figures apply to the case of infinite signal bandwidth in the receiver, implying that $R(\cdot)$ is a perfect triangular autocorrelation function.

For the purpose of model validation, the characteristic curves obtained while simulating the finite bandwidth case do not depart significantly from the infinite bandwidth case [6]. The $\tau_e$ term in (4) is determined by generating the multipath-distorted discriminator curve and finding the difference between the zero crossing that would be tracked by the DLL in a GPS receiver, and the zero crossing of the corresponding error-free discriminator curve.

In Figs. 3–6 the carrier-phase error envelopes for the coherent and the noncoherent code-tracking modes are characterized through simulation. It is to be noted that while the code tracking is performed in two different modes (coherent and noncoherent), phase lock is maintained on the composite signal by a carrier phase locked loop (PLL) in both modes.

The equation used to generate the code loop detector output for the coherent DLL in the simulation is

$$I_E - I_L$$

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where $I_E$ is the in phase value from the early correlator, and $I_L$ is the in phase value from the late correlator.

The normalization of the correlation function in the simulation rendered any normalization by the in-phase value of the prompt correlator superfluous. The code loop detector for the noncoherent DLL is

\[ \text{Early\_Power}^2 - \text{Late\_Power}^2 \]

with Early\_Power being the total power in the early component of the correlation function and, Late\_Power being the total power in the late component of the correlation function.

The computer based simulation implements a slightly different version of the code detector as compared with the real OEM3 (original equipment manufacturer) and OEM4 hardware used to obtain bench test data to help validate the theory. The OEM3 hardware implements a noncoherent dot product discriminator using the in-phase values and quad-phase values of the early, prompt, and late correlators in the receiver hardware. The OEM4 hardware was used to implement a coherent discriminator using only the in-phase values of the early, prompt, and late correlators. However, the results obtained in this section are valid for other implementations of the coherent and noncoherent code loop detectors.

In these plots, a simple scheme is used to differentiate between the coherent (solid line) and noncoherent (dotted line) cases. The envelope with the solid lines (inner envelopes) represents the maximum and minimum carrier-phase multipath errors for the coherent code-tracking mode. Similarly, the dotted lines (outer envelopes) represent the maximum and minimum carrier-phase multipath errors for the noncoherent code-tracking mode. The term “envelope” means the upper curve represents the maximum error bound and the lower curve represents the minimum error bound.

The standard correlator chip spacing (1 chip spacing) was modeled in Figs. 3 and 4. Fig. 3 demonstrates the carrier-phase multipath error envelope obtained for an M/D of $-3$ dB. Fig. 4 displays the results for an M/D of $-10$ dB.

It should be noted that in both cases (i.e., coherent and noncoherent), envelopes of the carrier-phase multipath error do not decrease uniformly with multipath delay. The slope of the envelope changes at approximately 0.7 chip. In addition, this anomalous characteristic at medium delay multipath is clearly seen only over the range of M/Ds where the strength of the multipath is a sizeable fraction of the direct (i.e., strong multipath environment).

Comparing Figs. 3 and 4, it is also apparent that the error envelope is wider (i.e., occupies a larger span of multipath delay) for higher M/D ratios. Fig. 5 depicts the carrier-phase error envelope for the narrow correlator (0.1 chip spacing) receiver architecture. The M/D ratio in this case is $-3$ dB. It can be observed that the medium delay characteristic of the carrier phase error envelope is markedly different from the case of the standard correlator. Specifically, the error envelope decreases more or less uniformly with multipath delay. The result for an M/D ratio of $-10$ dB is shown in Fig. 6.
Fig. 6. Carrier-phase multipath error envelope versus relative multipath delay for standard correlator spacing (1.0 chip) using (4). M/D = −10 dB. Figure depicts envelopes obtained for coherent (solid line) and noncoherent (dotted line) code-tracking modes.

It is to be noted that the carrier multipath envelopes depart from each other as a function of M/D ratio and correlator chip spacing. This departure is greater for higher M/D ratios and larger correlator spacing.

From the simulation results in Figs. 3–6 it is clear that the maximum carrier-phase multipath error for the strong multipath case departs from that of the weak multipath case. This is especially the case for the wide correlator architecture (1 chip spacing) as seen in Figs. 3 and 4. The departure is less obvious in the case of the narrow correlator spacing (0.1 chip) architecture. The coherent and noncoherent cases have almost identical carrier-phase error envelopes across the range of M/D ratios.

III. SIMPLIFIED MODELS

Previous researchers have developed simplified models to characterize carrier-phase multipath error. This section describes the limitations of these models. Equation (7) characterizes the maximum carrier-phase multipath error [7] as a function of relative amplitude of the multipath, code tracking error, and time delay of the multipath relative to the direct. This equation for the maximum carrier-phase multipath is

\[ \theta_c = \arcsin \left( \frac{\alpha R(\tau_c - \delta)}{R(\tau_c)} \right). \]  

(7)

This equation is valid when the multipath component of the signal is orthogonal to the composite signal. Simulating this equation involves estimation of the code tracking error and the phase of the multipath with respect to the direct that orients the multipath orthogonal to the composite. In order to find the code tracking error, knowledge of the multipath parameters (amplitude, delay and phase) is essential. Of these parameters, the simulation sets the amplitude and multipath delay. At this point, the phase of the multipath that maximizes the composite phase error is needed in order to obtain the corresponding code tracking error.

The requirement that the multipath should be orthogonal to the composite poses a restriction on the phase of the multipath with respect to the direct. However, this does not give the actual phase of the multipath (with respect to the direct) that meets the orthogonality requirement. In fact, the phase of the multipath with respect to the direct can be found only if the code tracking error is computed. The above argument throws some light onto the implementation complexities involving the simulation of (7).

Simplifications to (7) for maximum carrier phase multipath have been published [6]. In certain circumstances (i.e., narrow correlator) the code tracking error can be considered negligible. It is observed that the maximum carrier phase multipath error is obtained when this code tracking error goes to zero. As a result, \( \tau_c \) in (7) becomes zero. Exploiting the symmetric nature of the correlation function, the maximum carrier-phase multipath error is given by (8)

\[ \theta_c = \arcsin(\alpha R(\delta)). \]  

(8)

Implementing (8) in simulation yields envelopes for the maximum carrier-phase multipath error (Fig. 7). The M/D ratio is −3 dB. The multipath error envelope decreases uniformly with relative multipath delay.

A closer look at Fig. 7 would reveal that it is monotonic in nature and does not reveal any undulations in its envelope as a function of the correlator spacing and M/D ratio. It does not bring forth the undulation at medium delays, as predicted by (4) and seen in Figs. 3 and 4. This is due to the simplified nature of the model (8) used to generate Fig. 7.

IV. BENCH TEST SETUP

The first set of tests was performed to validate carrier-phase multipath theory for the noncoherent code tracking architecture. Zeta Associates Inc. performed bench tests and the results were provided.
to Ohio University for validation of theoretical multipath [8]. The next set of tests to validate carrier-phase multipath theory for the coherent code tracking architecture was performed by Ohio University at Honeywell Corporation’s AES division at Olathe Kansas. The fact that both sets of tests were not performed under the exact same conditions resulted in some minor differences between them. The theoretical basis behind the setups was the same. The setup to validate carrier-phase multipath for the noncoherent code tracking architecture is explained first. This is followed by the setup to validate carrier-phase multipath for the coherent code tracking architecture.

A. Noncoherent Code Tracking Architecture

A multi-channel GPS hardware simulator (Spirent/Northern Telecom STR2760) was used to generate a single multipath ray with specific amplitude and delay. Data was collected using a NovAtel OEM3 GPS receiver and included the time of measurement, pseudo-range, and carrier-phase. A special firmware load allowed the OEM3 receiver to operate in narrow and wide correlator tracking modes. This receiver employs a noncoherent code-tracking loop of the dot product type. Zero Doppler signals were produced by using a satellite simulated in a circular, zero inclination (equatorial) orbit. In a bench test environment, different sources of error as seen on the GPS signal can be turned off or on (except thermal noise). In this case, all sources of error except multipath were turned off.

The laboratory setup for the multipath tests consisted of a multi-channel GPS signal simulator, preamplifier, receiver and a computer to log the signal strength, pseudo-range, and phase data at a 1 Hz rate. One channel of the GPS signal simulator was configured to generate a direct path PRN-1 satellite signal at the ICD-GPS-200 specified minimum signal level (−130 dBm). A second channel generated a multipath signal with the same PRN code. This multipath signal was generated at a lower signal level compared with the direct and had a variable delay relative to the direct path signal [9].

Both standard (1-chip) and narrow (0.1-chip) correlator receivers were subjected to the same multipath scenarios. The tests were run for different multipath to direct power ratios (−2, −4, −7 and −10 dB). Multipath delay for each test was increased in fractional steps of a wavelength. The wavelength steps were $\lambda_{L1} = 1.25 \text{, } \lambda_{L1} = 1.25 \text{, } \lambda_{L1} = 1.375 \text{, } \lambda_{L1} = 1.5 \text{, } \lambda_{L1} = 28 \text{, } \lambda_{L1} = 28.125 \text{, } \lambda_{L1} = 28.25 \text{, } \lambda_{L1} = 2674.375 \text{, } \lambda_{L1} = 2674.5$. This scenario uses a delay step size that samples in-phase, out-of-phase, and at three intermediate points.

In order to allow receiver pseudo-range processing to settle at each multipath delay, the delay value was kept constant for a period of 20 s before changing to the next value (30 s dwell after a major step).

In addition to simulating a satellite with multipath (PRN-1), a second satellite was simulated with a range rate of 100 m/s (PRN-2). The carrier-phase data from this second satellite is used to remove simulator/receiver trends from the carrier-phase measurements on PRN-1 using the following expression

$$PRN1_{\text{corrected}} = PRN1_{\text{car}} - PRN2_{\text{car}} + (T * 100)/L1_{\text{wavelength}}$$ (9)

where

- $PRN1_{\text{corrected}}$ is the corrected carrier phase on PRN 1 in L1 cycles,
- $PRN1_{\text{car}}$ is the uncorrected carrier phase on PRN1 in L1 cycles,
- $PRN2_{\text{car}}$ is the uncorrected carrier phase on PRN2 in L1 cycles,
- $T$ is the time intervals between successive measurements,
- $L1_{\text{wavelength}}$ is the carrier wavelength at L1 frequency.

The receiver and simulator were phase locked to the same frequency reference. Data logged by the NovAtel receiver was in a proprietary format. This was converted to ASCII using NovAtel’s file conversion utility. Data with M/D ratios of −2 dB and −10 dB and different correlator spacing (standard and narrow) is used for the bench test comparison with the simulation results in order to capture the effects of strong and weak multipath.

B. Coherent Code Tracking Architecture

Bench tests were performed at the Aerospace Electronics Division of Honeywell at Olathe, Kansas in order to validate carrier-phase multipath theory for coherent code tracking receivers. A multi-channel GPS hardware simulator (Spirent STR4760) was used to generate a single multipath ray with specific amplitude and delay. Data was collected using a NovAtel OEM4 GPS receiver and included the time of measurement, pseudo-range, and accumulated carrier phase. A special firmware load allowed the OEM4 receiver to operate in narrow and wide correlator tracking modes. This special firmware load also employs a coherent code-tracking loop. In the previous bench setup an OEM3 receiver was used to collect data to validate the theory for the case of the noncoherent code tracking loop. In a bench test environment, different sources of error as seen on the GPS signal can be turned off or on (except inherent hardware error sources). Two PRNs with minimal cross correlation were used in the multipath testing procedure. The satellites transmitting these PRNs were colocated in a geo-stationary orbit with the
static user located at zero latitude, zero longitude and zero height. As a result zero Doppler signals were produced.

A second channel generated a multipath signal with the same PRN code. This multipath signal was generated at a lower signal level compared with the direct and had a variable delay relative to the direct path signal.

Both standard (1-chip) and narrow (0.1-chip) correlator receivers were subjected to the same multipath scenarios. The tests were run for different MDs (−3 and −10 dB). Multipath delay for each test was increased in fractional steps of a wavelength. The wavelength steps were \( \lambda_{L1} \), \( \lambda_{L1} \times 1.125 \), \( \lambda_{L1} \times 1.25 \), \( \lambda_{L1} \times 1.375 \), \( \lambda_{L1} \times 1.5 \), \( \lambda_{L1} \times 40 \), \( \lambda_{L1} \times 40.125 \), and \( \lambda_{L1} \times 40.25 \). This scenario uses a delay step size that samples in-phase, out-of-phase, and at three intermediate points at every 40 carrier cycles, corresponding to a resolution in the delay axis of approximately 7.6 m. In order to allow receiver pseudo-range processing to settle at each multipath delay, the delay value was kept constant for a period of 35 s before changing to the next value (60 s dwell after a major step).

A second satellite in the simulation was colocated in geo-stationary orbit with PRN-10 and transmitted PRN-20. It serves as a reference since it was not corrupted by multipath. The single difference of the receiver measurements made on these two satellites will remove any common error effects as seen on the GPS signal (i.e., atmospheric errors (if any) and common clock biases). The unnormalized cross correlation functions of GPS PRN codes have 3-levels (63, −1 and −65 relative to a peak autocorrelation value of 1023). However, there are combinations of PRN codes that have better cross correlation properties than others. For example, there are PRN combinations in which the cross correlation function near a relative offset (\( \tau \)) of zero are dominated by −1s rather than 63s and −65s. PRNs 10 and 20 have very low cross correlation sidelobes, (i.e., −1s) and were chosen for this reason.

Binary data was logged using the NovAtel receiver in its proprietary format. This was converted to ASCII using NovAtel’s file conversion utility. Data with M/D ratios of −3 dB and −10 dB and different correlator spacing (standard and narrow) are used for the bench test comparison with the simulation results in order to capture the effects of strong and weak multipath.

V. DATA ANALYSIS AND VALIDATION

Data collected through bench testing under controlled conditions is compared with the theoretical error envelopes obtained in Sections II and III. Such data proves to be extremely useful in validating the theoretical characterization of GPS carrier-phase multipath.

A single multipath ray was injected from the GPS hardware signal simulator and the receiver’s pseudo-range and carrier-phase measurements were logged to a file. Given zero Doppler in the configuration between GPS signal simulator and the receiver, there should not be any change to the measured pseudo-ranges and carrier-phase measurements over time.

Additional simulator/receiver trends were removed based on the measurements made on a second PRN that was transmitted without multipath added to it. As a result, any variation in the GPS carrier-phase measurements that are seen over the period of data collection is purely due to noise and multipath. The noise on the carrier phase measurement is in the millimeter range while the multipath is in the centimeter range.

A. Noncoherent Code Tracking Architecture

The noncoherent code-tracking mode was shown to affect the maximum carrier-phase multipath error envelopes quite significantly for the case of the standard correlator at high M/D ratios. This simulation result is verified with bench data in this section.

The first series of tests were conducted using the standard correlator architecture with the M/D ratios set at −2 dB and −10 dB. The plot of carrier-phase multipath error as a function of relative multipath delay can be seen in Fig. 8. The M/D ratio chosen for this test run was −2 dB. It is immediately obvious from Fig. 8 that envelope of the multipath error as inferred from the bench data is not a uniformly decreasing function of relative multipath delay. This confirms the results of the simulation seen in Fig. 3. In Figs. 8–11, the theoretical envelopes derived from (4) have been overlaid with the bench test data. The solid line represents the theoretically obtained carrier-phase multipath error envelope. The thickness of the solid line is an approximation of the 1\( \sigma \) noise bounds on the theoretical carrier-phase multipath error envelope.

Fig. 8. Carrier-phase multipath error versus relative multipath delay for standard correlator spacing, noncoherent code tracking. Bench test results overlaid with theoretical envelope (solid line), M/D = −2 dB.
Fig. 9. Carrier-phase multipath error versus relative multipath delay for standard correlator spacing, noncoherent code tracking. Bench test results overlaid with theoretical envelope (solid line), M/D = -10 dB.

When the M/D ratio is -10 dB, the nonuniform trend of the multipath error envelope seen in the -2 dB case is no longer clearly observable (Fig. 9). The next series of tests were conducted using the narrow correlator architecture. All other operating conditions were similar to the previous test run. The carrier-phase data collected with this narrow correlator architecture (0.1 chip) based GPS receiver is analyzed to observe the effects of multipath at different delays relative to the direct. The M/D ratio for Figs. 10 and 11 were -2 dB and -10 dB, respectively.

As observed in the figures, the error plots constitute a single-sided envelope. This is ascribed to the manner in which the multipath delay was incremented. The multipath delay ranges from nλ to (n + 0.5)λ and then steps to (n + 1)λ (n being a whole number).

As a result, the envelope does not manifest errors for multipath delays between (n + 0.51)λ to (n + 0.99)λ. These delays contribute to the other (positive) side of the multipath error envelope as seen in the simulations. Given the symmetric nature of carrier-phase multipath errors, it will suffice to analyze one-sided carrier-phase multipath error envelopes.

Upon comparison of the simulation results based on (8) and the bench test data, it is observed that this equation does not predict the nonuniformity in the multipath error envelope at medium delays for the standard correlator. This simplified expression for carrier-phase multipath error assumes that the code multipath error is very small and can be neglected. However, the code multipath error affects the estimation of the carrier-phase multipath error. Thus, (8) does not completely characterize carrier-phase multipath under all conditions. Specifically, for medium delay multipath, where the standard correlator experiences the largest code error, this simplified model is noticeably in error.

From Figs. 8–11 it is seen that the data collected through bench testing agrees quite well with the theoretical envelope overlaid upon the data (solid line). This theoretical envelope is a one-sided representation of the envelopes obtained from Figs. 3, 4, 5, and 7, which were generated based upon (4). The nonuniform characteristic in the carrier-phase envelope as predicted by (4) is also observed in the data collected through extensive bench tests. This is quite evident from Fig. 8. The simplified equations, due to their inherent assumptions, do not truly characterize the nature of carrier-phase multipath across all ranges of relative signal strengths and delays.

B. Coherent Code Tracking Architecture

The coherent code-tracking mode was shown to affect the carrier-phase multipath error envelopes appreciably for the case of the standard correlator at high M/D ratios. This simulation result will be verified with bench data in this section.

The first series of tests was conducted using the standard correlator architecture with the M/D ratios set at -3 dB and -10 dB. The plot of carrier-phase multipath error as a function of relative multipath delay can be seen in Fig. 12. The M/D ratio chosen for this test run was -3 dB. The envelope of the multipath error as inferred from the bench data is not a uniformly decreasing function of relative multipath...
delay. This confirms the results of the simulation seen in Fig. 3. In Figs. 12–15, the simulation data has been overlaid with the bench test data. The solid line represents the simulation data. The raw bench data has been averaged over each multipath delay value in order to minimize the effects of noise. As mentioned earlier, each delay value in the multipath profile was kept constant for a given period of time. Carrier-phase measurements from the latter half of each delay offset were used for the averaging procedure. Measurements from the first half of each delay offset in the profile were excluded from the averaging procedure, as they represent a transient response while stepping through the delay profile. Carrier-phase multipath error for each step in the delay profile is depicted by a single averaged value.

When the M/D ratio is $-10$ dB, the multipath error envelope is given by Fig. 13. The next series of tests were conducted using the narrow correlator architecture. All other operating conditions were effectively identical to the previous test run. The carrier-phase data collected with this narrow correlator architecture (0.1 chip) based GPS receiver is analyzed to observe the effects of multipath at different delays relative to the direct. Figs. 14 and 15 depict the comparison between theory and data for the narrow correlator (0.1 chip) with the M/D power ratios set at $-3$ and $-10$ dB, respectively.

As discussed for the previous test run, single-sided envelopes are provided. The excellent agreement between the theory and bench test results suggest that the more complete model presented as (4) is necessary for fully bounding carrier-phase multipath.

VI. HIGH FIDELITY MULTIPATH MODELING

The simulations used to derive the carrier-phase error envelope as shown in Figs. 12–15 were based on a theoretical model (4) that analyzed the interaction between the direct and multipath components of the GPS signals taking the amplitude, delay and phase of the multipath into account. In this section, a high fidelity receiver baseband processing model is used to complement the previous simulations. This high fidelity model implements the receiver code and carrier tracking loops and computes code and carrier multipath based on a truth reference. This model provides a holistic approach that incorporates the effects of noise and band limiting. Table I lists the
TABLE I
Attributes of Receiver Model

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<th>Attribute</th>
<th>Value</th>
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<tbody>
<tr>
<td>Front End BW (MHz)</td>
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<tr>
<td>Carrier Loop BW (Hz)</td>
<td>15 (2nd order PLL)</td>
</tr>
<tr>
<td>Carrier Aiding</td>
<td>yes</td>
</tr>
<tr>
<td>Code Loop BW (Hz)</td>
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<tr>
<td>Code Loop DLL</td>
<td>1 chip, dot product, normalized</td>
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<tr>
<td>Feedback Rates</td>
<td>5 Hz code/100 Hz carrier</td>
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<tr>
<td>PRN</td>
<td>10</td>
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<tr>
<td>MDR (Multipath-to-direct)</td>
<td>–3 dB/–10 dB</td>
</tr>
<tr>
<td>C/No (dB-Hz)</td>
<td>45</td>
</tr>
</tbody>
</table>

settings used for the receiver simulation that was implemented to characterize carrier-phase multipath.

In order to compare with the bench data, the simulation was run with multipath delay values defined as: \( \lambda \ast (40n, 40n + 0.125, 40n + 0.25, 40n + 0.375, 40n + 0.5) \) for \( n = 0 \) to 58, such that maximum delay is approximately 1.5 C/A code chips. Each step within a set whole number value of \( n \) is given a dwell time of 35 s. A settling time of 60 s is allowed when transitioning to a new set. Relative phase of the multipath signal, in this case, is anchored to the time delay of the multipath relative to the direct signal.

The results of this model are compared on a one-to-one basis with the bench test data in the following figures. It is to be kept in mind that these are raw phase measurements and have a higher noise level unlike the averaged values seen in the previous section. The trace in black represents the bench test data while the trace in gray represents the results of the receiver simulation. Fig. 16 depicts the wide correlator (1.0 chip spacing) coherent multipath scenario with M/D set at –3 dB. The bench data and the simulation results compare to a very good extent given the limitations imposed by noise. It can be observed that the nonlinearity in the carrier-phase error envelope seen in the bench test data is matched by the high fidelity multipath modeling. Fig. 17 depicts a zoomed-in version of Fig. 16. The black line represents the envelope of the bench test data in Figs. 16, 18, 20, and 22. The black dotted trace in Figs. 17, 19, 21, and 23 are used for the zoomed-in version of the plots and represents the bench data while the trace in gray represents the simulation data.

Fig. 18 depicts the wide correlator (1.0 chip spacing) coherent multipath scenario with M/D set at –10 dB. Fig. 19 depicts a zoomed-in version of Fig. 18. The black dotted trace is the bench data and the trace in gray represents the simulation data.

The next tests were performed for the narrow correlator (0.1 chip spacing) case with the multipath set at –3 and –10 dB, respectively. The results are shown in Figs. 20 and 22. Figs. 21 and 23 depict zoomed-in versions of Figs. 17 and 22, respectively. The black dotted trace is the bench data and the trace in gray represents the simulation data.

From Figs. 16, 18, 20, and 22 it is seen that the carrier-phase multipath error obtained from the high fidelity simulation compares vary favorably with the bench test results.

VII. ANALYSIS OF PHASE RATE EFFECTS

Relative motion of the satellite and/or receiver with respect to a reflecting surface will induce amplitude variation and non-zero phase rates on the multipath
component of the received signal. Receivers generally will reduce the effect of the multipath through effective averaging performed by the tracking loops [10]. One approach to characterize fading multipath is to sweep the multipath signal across the preset range of multipath delays at a given rate. In the scenario, this rate was set at an eighth of a wavelength per second. Such sweeping effectively causes a constant phase-rate resulting in multipath fading. Both wide and narrow correlator architectures were subject to the same conditions of fading and bench data was collected for the −3 dB and −10 dB cases. The receiver implementation included a carrier-aided coherent code-tracking loop.

Given the high degree of carrier aiding of the code-loop and extremely low code-loop bandwidth (approximately 0.05 Hz), it can be seen from Figs. 24 and 25 that much of the anticipated code multipath error is filtered out by the code loop due to the phase rate of the multipath which was set at 1/8 Hz. This rate was chosen in order to display the averaging effect of the pseudo-range multipath due to a slower code loop filter as mentioned above. Although the shape of the code error envelope is more or less preserved, the magnitude of the code multipath error as a function of relative multipath delay is extremely low in comparison to the static scenario [11]. This is discussed later in this section.

However, it is observed that the carrier-phase errors for the multipath fading scenario are very similar to the static multipath scenario. This similarity is attributed to the fact that carrier loop bandwidth (approximately 15 Hz) is much higher than the multipath fading frequency set at 1/8 Hz. This allows the carrier loop to keep track of the combination of the direct signal and the multipath. In such a case, the phase multipath errors do not get filtered out.
Figs. 24 and 25 pertain to the standard correlator coherent code-tracking mode. The multipath fading frequency is 1/8 Hz and the M/D ratio is $-3$ dB. The next set of figures (Fig. 26 and Fig. 27) pertain to the narrow correlator coherent code-tracking mode with the multipath fading frequency at 1/8 Hz and the M/D ratio set to $-3$ dB.

It is to be noted that the data displayed in Figs. 26 and 27 are for multipath delays between 0.1 and 1.5 chips. The data logging software in the receiver failed to log data between multipath delays of 0 to 0.1 chips.

From this data it is seen that the shape of the code range multipath error envelope is mostly preserved for both standard and narrow correlator cases. However, the peak value of these pseudo-range multipath errors is much less than in the static case for the given M/D ratios as seen in Figs. 28 and 29.

Fig. 28 depicts the pseudo-range errors for the narrow correlator strong multipath scenario with the multipath fading frequency at 1/8 Hz. The next set of figures (Fig. 26 and Fig. 27) pertain to the narrow correlator coherent code-tracking mode with the multipath fading frequency at 1/8 Hz and the M/D ratio set to $-3$ dB.
M/D ratio set at \(-3\) dB. The dotted envelopes depict the maximum multipath error for the static case as a function of time delay while the inner envelope shows the fading multipath error. The peak value for the standard correlator receiver architecture with the M/D ratio set at \(-3\) dB is about \(+/-110\) m in the static case. The peak error as seen from the fading multipath data is approximately \(+/-2\) m.

Fig. 29 depicts the pseudo-range errors for the narrow correlator strong multipath scenario with the M/D ratio set at \(-3\) dB. The dotted envelopes depict the maximum multipath error for the static case as a function of time delay while the inner envelope shows the fading multipath error. The peak value of the narrow correlator receiver architecture with the M/D ratio set at \(-3\) dB is about \(+/-11\) m in the static case. The peak error as seen from the fading multipath data is approximately \(+/-0.5\) m.

The code DLL is unable to follow the dynamics of the multipath fading and essentially filters out the pseudo-range multipath. This is due to the extremely narrow code loop bandwidth allowed when using a high degree of carrier aiding in the code loop. The carrier tracking loop bandwidth is wide enough to accommodate significant line of sight Doppler and Doppler rates. This design aspect of the receiver effectively enables the carrier-tracking loop to follow the 1/8 Hz fading rate. As a result, the carrier-phase tracking error is not mitigated through loop bandwidth filtering as in the pseudo-range multipath case. These can be seen from Figs. 30 and 31. Figs. 30 and 31 compare the carrier-phase multipath error obtained for the static case with the carrier-phase error from the fading case. The dotted black lines depict the envelope of the carrier-phase multipath error for the static case. The noisy black profile depicts the carrier-phase multipath error for the fading scenario. It is easily seen that a fading bandwidth of 1/8 Hz is very low compared with the bandwidth of the carrier-tracking loop and does little to affect the carrier-phase multipath error.

The theoretical carrier-phase multipath error envelopes obtained for the coherent and noncoherent code tracking GPS receiver have been validated through bench tests. The validity of two simplified carrier-phase multipath models (7) and (8) have been investigated. Carrier-phase multipath theory agrees well with the bench test data for the complete KALYANARAMAN ET AL.: CODE TRACKING ARCHITECTURE INFLUENCE ON GPS CARRIER MULTIPATH 559

IV. CONCLUSION

A typical example of fading multipath would be an airborne receiver picking up ground reflection multipath. The following equation [12] is used to compute the frequency difference between the direct and the multipath signal (ground reflection) reaching the airborne user from the satellite vehicle

\[
f_{\text{diff}} = \frac{\Delta p}{c} \times f_{\text{nom}}
\]

where

- \(f_{\text{diff}}\) is the difference frequency between the direct and the multipath signal,
- \(f_{\text{nom}}\) is the nominal frequency of the GPS signal,
- \(c\) is the speed of light in m/s,
- \(\Delta p\) is the time derivative of the path length difference between the direct and the reflected (multipath) signal from the satellite vehicle.

\[
d(\Delta p)/dt \leq 2.5 \text{ m/s}
\]

\[
\Delta p = 2 \times h \times \sin(\alpha)
\]

where \(h\) is the height of the GPS user above the ground, and \(\alpha\) is the angle of elevation of the satellite vehicle at the user and the ground reflection point (assuming a plane wave incidence for the direct and multipath components of the signal).

For a typical aircraft approach and landing scenario, the “sink rate” (i.e., derivative of aircraft height, \(h\)) is approximately 2.5 m/s. This yields, via (10) and (11), frequency differences in the range of 2–20 Hz for satellite elevation angles in the range of 5 to 90 deg [12].

VIII. CONCLUSION

The theoretical carrier-phase multipath error envelopes obtained for the coherent and noncoherent code tracking GPS receiver have been validated through bench tests. The validity of two simplified carrier-phase multipath models (7) and (8) have been investigated. Carrier-phase multipath theory agrees well with the bench test data for the complete
theoretical model (4). The existence of an anomalous characteristic in the multipath error envelope, as predicted by (4), has been verified experimentally for the coherent and the noncoherent code tracking architectures. This corroborates the model utilized for the theoretical characterization of carrier-phase multipath (4). The effect of fading on code and carrier multipath errors has been investigated for the case of the coherent code-tracking loop implementing wide and narrow correlator spacing. In addition, the utilization of carrier-aided code loops to mitigate the effects of fading multipath on pseudo-range accuracy has been demonstrated, and the tendency of signal dynamics to reduce code multipath substantially more than carrier multipath has also been shown.

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REFERENCES
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