Analysis of Passive Suspension System using MATLAB, Simulink and SimScape

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Abstract
The purpose of the suspension system in automobiles is to improve ride comfort and road handling. In this current work the ride and handling performance of an automobile with passive suspension system is simulated and analyzed. The suspension system modeled considered here as a two degree of freedom quarter car model. The suspension system designed here is totally designed and analyzed separately using MathWorks products namely MATLAB, Simulink and Simscape and tries to draw conclusion on which of these MathWorks products provides better results and which product is easy to work with, in relation lumped parameter models in this case suspension system

1.1 Introduction
Traditionally automotive suspension designs have been a compromise between the three conflicting criteria of road holding, load carrying and passenger comfort. The suspension system must support the vehicle, provide directional control during handling maneuvers and provide effective isolation of passengers/payload from road disturbances. Though a passive suspension system has the ability to store energy via a spring and to dissipate it via a damper. Its parameters are generally fixed, being chosen to achieve a certain level of compromise between road holding, load carrying and comfort. [1]

The work presented here tries to analyze the effect of seat suspension on vehicle performance of a quarter car model for a given road input using different approaches namely analysis by using state space equations in MATLAB, analysis by equation of motion using mathematical blocks available in Simulink and finally through physical modeling using Simscape blockset library.
1.2 Problem Description:

Consider a passive suspension system of a quarter car model as shown in figure below.

Where $Z_r$, $Z_u$, $Z_s$ and $Z_{se}$ are the vertical displacement of road, unsprung mass, sprung mass and seat. Simulate the output response for sudden change in road profile of 0.2 m height and draw the conclusion.

Model parameters are:

Mse-seat and driver mass (90Kg)
Ms-is the quarter of the vehicle sprung mass (250Kg)
Mu-is the quarter of the vehicle unsprung mass (40Kg)
bs- Damping ratio of the vehicle suspension (2000Ns/m)
bse- Damping ratio of the seat suspension (3000Ns/m)
Kt=Tire stiffness (125000N/m)
Ks-Vehicle suspension spring stiffness (28000N/m)
Kse-seat suspension spring stiffness (8000N/m)
Zr=0.2 Road input (m)

1.3 Quarter model of Passive Suspension System

The governing equations of the quarter car suspension system (Figure 1.1) are presented. The free body diagram for the car model is shown in Figure 1.2

![Free body diagram of the passive suspension system](image)

Figure 1.2 Free body diagram of the passive suspension system

1.4 Equation of Motion:

Equation of motion from Fig 1.2 for drive and seat mass is given as

\[ M_{se} \ddot{Z}_{se} + K_{se} (Z_{se} - Z_s) + b_{se} (\dot{Z}_{se} - \dot{Z}_s) = 0 \]

Or
Equation of motion for sprung mass from Fig 1.2

\[ \ddot{Z}_{se} = -K_{se} (Z_{se} - Z_s)/M_{se} - b_{se} (\dot{Z}_{se} - \dot{Z}_s)/M_{se} \quad \text{--- Eq 1} \]

\[ \begin{align*}
M_{s} \ddot{Z}_{s} - K_{se} (Z_{se} - Z_s) - b_{se} (\dot{Z}_{se} - \dot{Z}_s) + K_{s} (Z_s - Z_u) + b_{s} (\dot{Z}_s - \dot{Z}_u) &= 0 \\
\dot{Z}_{s} &= \frac{K_{se} (Z_{se} - Z_s)}{M_{s}} + \frac{b_{se} (\dot{Z}_{se} - \dot{Z}_s)}{M_{s}} - \frac{K_{s} (Z_s - Z_u)}{M_{s}} - \frac{b_{s} (\dot{Z}_s - \dot{Z}_u)}{M_{s}} \quad \text{--- Eq 2} \\
\end{align*} \]

Similarly from Fig 1.2 equation of motion for unsprung mass is

\[ \begin{align*}
M_{u} \ddot{Z}_{u} - K_{s} (Z_s - Z_u) - b_{s} (\dot{Z}_s - \dot{Z}_u) + K_{i} (Z_u - Z_r) &= 0 \\
\dot{Z}_{u} &= \frac{K_{s} (Z_s - Z_u)}{M_{u}} + \frac{b_{s} (\dot{Z}_s - \dot{Z}_u)}{M_{u}} - \frac{K_{i} (Z_u - Z_r)}{M_{u}} \quad \text{--- Eq 3} \\
\end{align*} \]

1.5 State Space Equation

From Eq1, Eq2 and Eq3 state space equation can be derived by proper assumption of state variables.

Let us assume state space variable are:

\[ \begin{align*}
x_1 &= \dot{Z}_{se} ; x_2 = Z_s - Z_{se} ; x_3 = \dot{Z}_s ; x_4 = Z_u - Z_s ; x_5 &= \dot{Z}_u ; x_6 = Z_r - Z_u \quad \text{--- Eq 4} \\
\end{align*} \]
Substituting these space variables in Eq 1, Eq2 and Eq3, the space state equation can be written as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-M_{se}\frac{K_{se}}{M_{se}} & -b_{se} & M_{se}\frac{K_{se}}{M_{se}} & -b_{se} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
M_{s}\frac{K_{se}}{M_{s}} & M_{s}\frac{b_{se}}{M_{s}} & -M_{s}\frac{K_{se}}{M_{s}} & -K_{s} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
u
\end{bmatrix}
\]

Using eq 4, output equation for displacement and velocity output equation can be derived as

\[
y =
\begin{bmatrix}
Z_{se} \\
Z_{s} \\
Z_{u} \\
\dot{Z}_{se} \\
\dot{Z}_{s} \\
\dot{Z}_{u}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
u
\end{bmatrix}
\]

1.6 Analysis of the suspension system using MATLAB

As MATLAB® is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation and Control System Toolbox software provides tools for systematically analyzing, designing, and tuning linear control systems. The suspension system is modeled using the state space equation derived above and with help of the functions available in MATLAB and Control System toolbox [2].

Refer to the Suspension_sys_MATLAB.m file for source code.

1.7 Analysis of Suspension system using Simulink
Using the equations Eq1, Eq2 and Eq3, the simulink model of the suspension system is designed. The model consists of the basic blocks from the simulink library. Figure 1.3 represents the simulink model for the suspension system.

![Figure 1.3 Suspension system](image)

### 1.8 Analysis of Suspension System using Simscape

Simscape™ extends Simulink® with tools for modeling and simulating multidomain physical systems, such as those with mechanical, hydraulic, and electrical components. Simscape can be used for a variety of automotive, aerospace, defense, and industrial-equipment applications [2].

With Simscape you build a model of a system just as you would assemble a physical system. This approach lets you describe the physical structure of a system rather than the underlying mathematics. From your model, which closely resembles a schematic, Simscape automatically constructs equations that characterize the behavior of the system. These equations are integrated with the rest of the Simulink model.
Using the blocks available in SimScape library, the Figure 1.4 shows the snapshot of suspension system modeled using Simscape blockset.

Figure 1.3 Suspension system using SimScape.

1.9 Conclusion

MATLAB, Simulink and Simscape software were similar and results obtained were same, when performing lumped parameter modeling in this suspension system. But building the suspension model using SimScape was much easier when compared with MATLAB code or Simulink blocks. However modeling with MATLAB and Simulink was more flexible.
With respect to the suspension system, it is can be concluded that the driver does experience vibration during normal operations under passive suspension system may not be as much as with respect to the sprung mass or unsprung mass, but significant to cause an effect on drier’s health.

Bibliography

