ZHAO Yi, ZENG Ling-kang, XIE Gang, LIU Yuan-an, XIONG Fang

Fairness based resource allocation for uplink OFDMA systems

Abstract This article investigates two fairness criteria with regard to adaptive resource allocation for uplink orthogonal frequency division multiple access (OFDMA) systems. Nash bargaining solution (NBS) fairness and proportional fairness (PF) are two suitable candidates for fairness consideration, and both can provide attractive trade-offs between total throughput and each user’s capacity. Utilizing Karush-Kuhn-Tucker (KKT) condition and iterative method, two effective algorithms are designed, to achieve NBS fairness and proportional fairness, respectively. Simulation results show that the proposed resource allocation algorithms achieve good tradeoff between the overall rate and fairness, with little performance loss from the total capacity.

Keywords OFDMA, resource allocation, fairness

1 Introduction

Driven by the increasing popularity of wireless broadband services, the next-generation mobile communication systems will undergo a rapid growth of high-data-rate applications with diverse quality of service requirements. To support such applications under limited radio resources and harsh wireless channel conditions, OFDMA has emerged as one of the most promising multiple access techniques, because it inherits the advantages of orthogonal frequency division multiplexing (OFDM), such as, transforming the frequency selective fading channel into a large number of flat fading subchannels and providing more flexibility for future multimedia services.

For the efficient utilization of scarce radio resources, adaptive resource allocation has recently been one of the most important issues [1–7]. However, most of the existing resource allocation algorithms mainly focus on finding a solution for the minimization of the overall transmission power under the data rate constraint [1, 2] or the maximization of the total data rate under a power constraint [3, 4]. Utilizing the KKT condition, Reference [4] proposes a joint subcarrier and power allocation algorithm for uplink OFDMA systems, to maximize the overall capacity. Via the greedy subcarrier allocation algorithm, based on marginal rate function and iterative power allocation algorithm, the proposed algorithm in Ref. [4] achieves a near optimal solution. When the path loss differences among users are great, it is possible that the user with a higher average channel gain will dominate most resources, that is, subchannels and power. As those algorithms offer no mechanism to enhance their competitive ability to get resource from good users, the users with lower average channel gains may be unable to receive any service. Therefore, fairness has been an important component in designing adaptive resource allocation schemes. Fairness is a loose concept, which implies that all users get an equitable amount of system resources to maintain the requirement of quality of service (QoS).

Game theory is a powerful tool for analyzing the interaction of decision makers with conflicting objectives, which can provide the crucial notion of fairness and maximize the overall system rate. Nash equilibrium is the most important equilibrium concept in the game theory, which shows a stable tradeoff between different bargainers. Reference [8] draws upon the Nash bargaining framework, from the cooperative game theory, and provides precise mathematical characterization of the solutions and their properties. On the basis of the NBS fairness criterion, a novel algorithm has been developed in Ref. [5], which mitigates the tension of total capacity and fairness in uplink OFDMA systems. The NBS fairness criterion can provide a better performance in the total capacity, and the total capacity is distributed more fairly and rationally among users than the sum capacity maximization method. However, the rate ratio of the users undergoing random wireless fading channel is uncontrollable, as the rate ratio of users under NBS fairness only relates to their channel condition realization.

In most practical wireless systems, different users require different data rates to accomplish different classes of service. To provide flexible rate control, another important QoS guarantee fairness criterion—proportional fairness (PF) criterion is employed.
in terms of predetermined rate ratios, to assure each user to achieve the target data rate. In Ref. [9], Viswanath et al. discuss long-term proportional fairness resource allocation with “dumb” antennas. In Ref. [10], Mohanram C and Bhashyam S proposed an efficient subcarrier and power allocation algorithm for downlink multiuser OFDM systems, which allocate each subcarrier iteratively, to maximize the total rate with constraints of a set of predetermined proportional fairness ratios.

In this article, the authors investigate the fairness issue for uplink OFDMA systems based on NBS fairness and PF criteria. The algorithm in Ref. [5] achieves the final results in an iterative way, but it cannot be applied to real-time communication. There is much to be pessimistic about, as this algorithm sometimes cannot converge to the optimal solution because of the nonlinear and combinatoral nature of the formulated problem. To overcome the disadvantages in Ref. [5], the authors have developed a new algorithm, which can achieve NBS fairness effectively by using the KKT condition. In contrast to [9], where proportional fairness is maintained for long time allocation, the authors have proposed a PF algorithm to maintain proportional rates among users for each channel realization, which ensures the rates of different users to be proportional in any time scale of interest. The PF algorithm in Ref. [10] is designed for the downlink OFDM system. Here, the novel PF algorithm is proposed for the uplink OFDMA systems and develops an iterative method to solve the PF problem.

The rest of this article is organized as follows. Section 2 outlines the system model of an uplink OFDMA system, and gives statements of all the problems. The proposed resource allocation algorithms for NBS fairness and PF fairness are described in Sect. 3 and Sect. 4, respectively. To demonstrate the potential of the proposed algorithms, analysis along with Monte Carlo simulation results are provided in Sect. 5. Finally, the conclusions are drawn in Sect. 6.

2 System model

The authors consider an uplink scenario of a single-cell OFDMA system equipped with $N$ subcarriers and $K$ users. Each user experiences different channel gains on each subcarrier and is assumed to be able to estimate the channel state information perfectly. The channel state information (CSI) is the feedback to the base station (BS) via a separate channel without any time delay. According to a given scheduling criterion, BS allocates the resource to each user as soon as the CSI is collected and informs each user. In this article, the authors assume that the transmitted signals experience a slow, time-varying fading channel, thus the channel coefficients can be regarded as constant during the period of subcarrier allocation and power loading. The bandwidth of each subcarrier is also assumed to be much smaller than the coherent bandwidth of the channel, which insures that the channel gain for each subcarrier is constant over its bandwidth.

The structure of the uplink OFDMA system under consideration is shown in Fig. 1. At BS, with feedback CSI, resource allocation assignment is implemented by the resource scheduling module and the scheduling results are sent to each user. At MS, each user’s data are fed into the encoder. After the process of encoding, inverse fast Fourier transform (IFFT), parallel to serial (P/S) conversion and adding cycle prefix (CP) data are transmitted. Making use of the scheduling results, the reverse operation is done at the BS to recover the data of the corresponding users.

![Fig. 1 Block diagram of a uplink OFDMA system with adaptive resource allocation](image)

Let $h_{i,j}$ denote the channel gain at the $j$th $(1 < j < N)$ subcarrier of the $i$th $(1 < i < K)$ user. $p_{i,j}$ denotes the allocated power of user $i$ on subcarrier $j$ and $\sigma^2$ denotes the variance of additive white Gaussian noise. Then, the capacity for user $i$, denoted by $R_i$, is defined as:

$$R_i = \sum_{j=1}^{N} a_{i,j} r_i = \sum_{j=1}^{N} a_{i,j} \log_2 \left(1 + \frac{p_{i,j} |h_{i,j}|^2}{\sigma^2} \right)$$

(1)

where $r_{i,j}$ denotes the $i$th user’s transmission rate on the $j$th subcarrier. Although only one total power constraint at the base station exists for downlink, multiple total power constraints exist for the uplink depending on the number of users, that is, $\sum_i p_{i,j} = 1$, $\forall$ user $i$. $a_{i,j}$ is a subcarrier allocation indicator, which is defined as follows: $a_{i,j} = 1$ if subcarrier $j$ is allocated to user $i$, otherwise $a_{i,j} = 0$. For the sake of simplicity, each subcarrier is assumed to support one user for a certain time, that is, $\sum_{i=1}^{K} a_{i,j} = 1$ $\forall$ user $i$. $\Gamma$ is the well-known
“signal to noise ratio (SNR) gap”, for quadrature amplitude modulation (QAM), \( \Gamma \) has a simple relationship with the required bit error rate \( P_{\text{BER}} \) \text{[11]}: \( \Gamma = - \ln(P_{\text{BER}})/1.5 \)

Let \( \Omega \) denote the set of channel indices assigned to user \( i \). As each subcarrier can only belong to one user exclusively, at each slot, all \( \Omega_i \), \( i = 1, 2, ..., K \) should be disjointed. For simplicity, the bandwidth of each subcarrier is normalized to 1 and the maximum allowable transmission power of each user is also constrained to 1 regardless of how many subcarriers are allocated to the user.

From [3], the optimizing function of NBS fairness can be expressed as follows:

\[
\max U_{\text{NBS}} = \max_{\omega_{i,j}} \prod_{i=1}^{K} R_i = \max_{\omega_{i,j}} \prod_{i=1}^{K} \sum_{j=1}^{N} \omega_{i,j} \log_2 \left( \frac{p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right)
\]

subject to:

\[
\sum_{j=1}^{N} \omega_{i,j} \leq 1 \quad \text{for all } j
\]

\[
\sum_{j=1}^{N} p_{i,j} \leq 1 \quad \text{for all } i
\]

\[
p_{i,j} \geq 0, \omega_{i,j} \geq 0 \quad \text{for all } i, j
\]

\[\Omega_i \cup \Omega_j \cup \cdots \cup \Omega_K = \{1, 2, ..., N\}\]

\[\Omega_i \cap \Omega_j = \phi; \quad i, j = 1, 2, ..., K, \ i \neq j\]

In Ref. [10], the objective function of PF is to maximize the downlink throughput, although maintaining a predetermined ratio among each user’s rate. For the uplink scenario, the optimization problem is formulated as:

\[
\max U_{\text{PF}} = \max_{\omega_{i,j}} \sum_{i=1}^{K} R_i = \max_{\omega_{i,j}} \sum_{i=1}^{K} \sum_{j=1}^{N} \omega_{i,j} \log_2 \left( \frac{p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right)
\]

subject to:

\[
\sum_{j=1}^{N} \omega_{i,j} \leq 1 \quad \text{for all } j
\]

\[
\sum_{j=1}^{N} p_{i,j} \leq 1 \quad \text{for all } i
\]

\[
p_{i,j} \geq 0, \omega_{i,j} \geq 0 \quad \text{for all } i, j
\]

\[\Omega_i \cup \Omega_j \cup \cdots \cup \Omega_K = \{1, 2, ..., N\}\]

\[\Omega_i \cap \Omega_j = \phi; \quad i, j = 1, 2, ..., K, \ i \neq j\]

where \( \{\alpha_1, \alpha_2, ..., \alpha_K\} \) is a set of predetermined values that are used to ensure proportional fairness among users.

### 3 Resource allocation for NBS fairness

The problem formulated in Eq. (3) can be regarded as an integer programming (IP) problem, and the mathematical theory has not found the effective way to solve IP with more variables and constraints. To make the problem tractable, \( \omega_{i,j} \) is relaxed to be a real number between 0 and 1 \([12, 13]\), and KKT conditions for optimality are adopted to analyze the characteristic of the optimal NBS solution with inequality constraint conditions.

Using Lagrangian relaxation, the NBS fairness cost function can be expressed as:

\[
U_{\text{NBS}}(\omega_{i,j}, p_{i,j}) = -\log \left( \prod_{i=1}^{K} \sum_{j=1}^{N} \omega_{i,j} \log_2 \left( \frac{1 + p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right) \right) + \sum_{j=1}^{N} \lambda_j \left( \omega_{i,j} - 1 \right) + \sum_{j=1}^{N} \mu_j \left( \sum_{i=1}^{K} p_{i,j} - 1 \right)
\]

Taking the derivatives with respect to \( \omega_{i,j} \) and considering the KKT condition, the authors can get:

\[
\frac{\partial f}{\partial \omega_{i,j}} = -\frac{\lambda_j}{\log_2 \left( \frac{1 + p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right)} + \lambda_j = 0
\]

\[
\sum_{j=1}^{N} \omega_{i,j} \leq 1 \quad \text{for all } i
\]

\[
\sum_{j=1}^{N} p_{i,j} \leq 1 \quad \text{for all } i
\]

\[
p_{i,j} \geq 0, \omega_{i,j} \geq 0 \quad \text{for all } i, j
\]

\[\Omega_i \cup \Omega_j \cup \cdots \cup \Omega_K = \{1, 2, ..., N\}\]

\[\Omega_i \cap \Omega_j = \phi; \quad i, j = 1, 2, ..., K, \ i \neq j\]

\[
\lambda_j \left( \sum_{i=1}^{K} p_{i,j} - 1 \right) = 0
\]

\[
\lambda_j \left( \sum_{i=1}^{K} \omega_{i,j} - 1 \right) = 0
\]

From Eq. (6) and Eq. (9), if the subcarrier \( j \) is not allocated to user \( i \) (that is, \( \omega_{i,j} = 0 \)), then \( -\log_2 \left( \frac{1 + p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right) / R_i + \lambda_j \leq 0 \), and if subcarrier \( j \) is allocated to user \( i \) (that is, \( \omega_{i,j} > 0 \)), then \( -\log_2 \left( \frac{1 + p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right) / R_i + \lambda_j = 0 \). This means that subcarrier \( j \) should be allocated to user \( i \):

\[
\log_2 \left( \frac{1 + p_{i,j} |h_{i,j}|^2}{\sigma^2 \Gamma} \right) / R_i + \lambda_j = 0
\]

Equation (10) is a necessary condition for the optimal
solution, which gives the character of the optimal solution of NBS fairness cost function. Getting rid of the variable $\omega_j$ has explicit physical meaning: the subcarrier should be allocated to the user who has the largest portion of the total rate among all the users on the same subcarrier. On the basis of this explicit physical meaning, the authors construct a similar method with [4]:

**Initialization:** $\Omega = \{1, 2, \ldots, N\}, \Omega' = \emptyset, \forall i, i \in \{1, 2, \ldots, K\}.$

**Step 1** Assume all unallocated subcarriers $j$ are allocated to each user $i$, calculate $P_{ij}$ by using the water filling method in Ref. [14].

**Step 2** Select the maximum value:

$$ (i', j') = \arg \max_{i,j} \frac{\log_2 \left( 1 + \frac{P_{ij} |h_{ij}|^2}{\sigma^2} \right)}{\sum_j \omega_j \log_2 \left( 1 + \frac{P_{ij} |h_{ij}|^2}{\sigma^2} \right) } $$

**Step 3** $j' \in \Omega'$ and $\Omega = \Omega - \{j'\}.$

**Step 4** Repeat Step 1, Step 2, and Step 3 until all subcarriers are allocated.

**Step 5** Water filling all the allocated subcarriers for each user, to get the final power allocation.

### 4 Resource allocation for proportional fairness

As the optimal solution to the proportional fairness is computationally very complex to obtain, a low-complexity suboptimal algorithm is expected. Following a similar thought as [10], the authors have constructed a suboptimal solution to the optimization problem in Eq. (4).

To deal with the multivariable optimization problem, equal power to each subcarrier is assumed to reduce the number of free variables [10]. However, this assumption cannot be applied to the uplink case because of the distributed power constraints of each user.

The joint subcarrier and power allocation strategy is as follows:

**Initialization:** $\Omega = \{1, 2, \ldots, N\}, \Omega' = \emptyset, \forall i, i \in \{1, 2, \ldots, K\}.$

**a)** Assume that each user takes up all the subcarriers, and allocates power to subcarriers using the water filling method for each user.

**b)** $\forall i$, do $j_i = \arg \max_{j \in \Omega} (p_{ij} |h_{ij}|^2) / (\sigma^2 \Gamma), \Omega' = \{j_i\}, \Omega = \Omega \cup \{j_i\},$ $R_i = \log_2 \left( 1 + \frac{P_{ij} |h_{ij}|^2}{\sigma^2 \Gamma} \right), \Omega = \Omega - \{j_i\}.$ If the other users choose the same subcarrier $j'$, then the authors will allocate $j'$ to the user with the largest $\alpha$, and the other users have to choose another subcarrier with the second largest $p_{ij} |h_{ij}|^2 / (\sigma^2 \Gamma)$.

**5 Simulation results**

To evaluate the performance of the proposed algorithms, the authors compare them with the max-rate algorithm. The result of the algorithm in Ref. [4] is regarded as an upper bound for total capacity. The simulation results are given by the Monte Carlo method using 6-path frequency selective Rayleigh fading channel. The channel parameters are presented in Table 1. Other simulation parameters involved within the simulation process are shown in Table 2.

**Table 1** Parameters of 6-path Rayleigh fading channel model

<table>
<thead>
<tr>
<th>Path</th>
<th>Delay/μs</th>
<th>Power/dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>1.6</td>
<td>-6</td>
</tr>
<tr>
<td>5</td>
<td>2.3</td>
<td>-8</td>
</tr>
<tr>
<td>6</td>
<td>5.0</td>
<td>-10</td>
</tr>
</tbody>
</table>

**Table 2** Statistical simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth (normalized)</td>
<td>1</td>
</tr>
<tr>
<td>Number of subcarriers</td>
<td>16–128</td>
</tr>
<tr>
<td>Maximal power for each user</td>
<td>1</td>
</tr>
<tr>
<td>Noise variance</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>BER</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Number of users</td>
<td>2–16</td>
</tr>
</tbody>
</table>

Figures 2 and 3 show the capacity and rate product versus the number of users when the system has 128 subcarriers. All
users’ locations are randomly generated and distributed uniformly within the range of 40 m to 200 m. PF(NBS) indicates that the rate ratio of the PF algorithm is the same as that of NBS, and PF(1:1:1:1) denotes the rate ratio of the PF algorithm to be 1:1:1:1. The performances of NBS and PF(NBS) are between the max-rate and PF(1:1:1:1). Figure 2 demonstrates that the performance of PF(NBS) is slightly worse than that of NBS. With the number of users increasing, the performance gap between the max-rate and fairness algorithms (both NBS and PF) becomes larger. Figure 3 shows the product of each user’s rate (dB), which is the fairness index of NBS, versus the number of users.

As the authors had expected, the NBS had the largest value. The max-rate showed the worst performance, especially when the system was heavily loaded, as the increasing number of users in the system enhanced the probability that some users experienced deep fading. To avoid the case where the product of the max-rate became zero, the authors defined a new fairness index \( f_{\text{index}} \) as: 
\[
 f_{\text{index}} = 10 \log_{10} \left( 1 + \prod_{i} R_{i} \right) .
\]

Figures 4 and 5 show the performance of PF versus the number of subcarriers. It is assumed that four users with the same average channel gain are involved in a system. The rate ratio of PF is configured as 1:1:1:1, 1:1:2:2, and 1:2:4:8. From Fig. 4, the authors can see, PF(1:1:1:1) is adjacent to the upper bound tightly, however, PF(1:2:4:8) has the largest capacity loss, as it deviates the average rate ratio of the max-rate more severely. The authors have reached an intuitive conclusion that the further the rate ratio of PF deviates, the larger is the capacity loss. Figure 5 presents the normalized rate ratio square error (NRRSE) performance of the PF algorithm with different configurations. The dB of NRRSE is defined as:
\[
 V_{\text{NRRSE}} = 10 \log_{10} \left( \sum_{k} \left( R_{\text{q}} / R_{i} - \alpha_{k} \right)^{2} \right),
\]
where \( k \) denotes the index of users. PF(1:1:1:1) achieves the predetermined rate ratio easier.

6 Conclusions

In this article, the authors compare two fairness criteria in the uplink OFDMA systems: the NBS fairness criterion and the PF criterion. The former shows a better performance in total capacity, but the BS cannot control the rate ratio because it only depends on the channel state of users. The latter criterion can provide a controllable rate ratio regardless of the channel condition for each user. Nevertheless, to achieve the hard fairness, the system capacity degrades sharply. If the authors configure the same rate ratio for PF criterion as NBS,
the capacity of the two strategies come reasonably close to each other.

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References


Biographies: ZHAO Yi, Ph. D. Candidate, Ministry of Education Key Laboratory of Universal Wireless Communications, Beijing University of Posts and Telecommunications, interested in the research of cross-layer optimization and adaptive resource allocation algorithms for MIMO-OFDMA systems, handover schemes for multi-cell systems, HARQ schemes.

LIU Yuan-an, Ph. D., Ministry of Education Key Laboratory of Universal Wireless Communications, of Beijing University of Posts and Telecommunications, professor, interested in the research on mobile communication and electro-magnetic compatibility (EMC).