Research Of Multi-Dimension Assignment Algorithm Of Data Association For Passive-Sensor Location System

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ABSTRACT

Direction-finding cross location algorithm in multi-passive-sensor location systems is a key problem. This paper studies the problem and proposes a modified multi-dimension (S-D) assignment algorithm to solve it. In order to find the global optimal solution, calculation of cost function of all intersection points of passive sensors must be considered, so the calculation burden is heavier. The algorithm proposed in this paper uses a method of passive cross location to eliminate some false intersection points, so that the computation burden of calculating the cost function is decreased. In the meantime, with the removing of the large number of false location points, the effect of data association can be improved compared with the case of calculating the global costs. Simulation results verify the feasibility and validity of the algorithm presented in this paper.

1. INTRODUCTION

Direction finding location system has the advantage of high concealment and can get a lot of properties of targets[1,2]. When using passive location methods to track targets, the existence and anti-interference capability of the passive location systems can be improved. Compared with active radar, passive radar possess the potential capability of detect stealth air vehicle, low altitude target and anti radar missile. Now passive-sensor location has become a hot point which is paid much more attention by many scholars. When the targets positions are estimated by bearing-only measurements, the better method is to transform the data association problem to the optimal assignment problem of operational research. The disadvantage of the optimal assignment algorithm of multi-passive-sensor multi-target is the long processing time for the calculation of association cost. To counter this problem, a method is proposed in this paper. Aiming at the different cases that four or three bearing-only sensors and multi-target are in the same plane, this paper proposes a modified algorithm based on direction-finding cross location respectively. The essence of the modified algorithm is a two-stage association algorithm. In the first association process, we use a cross location algorithm to eliminate some false intersection points, so the computation burden of calculating the cost function of assignment problem is decreased greatly. In the meantime, with the removing of the large number of false location points, the effect of data association is improved in the assignment process, i.e., the second association process. Simulation experiments aiming at different cases of 3-D and 4-D assignment problem are designed to verify the effectiveness of the new algorithm, and results of comparison and analysis of different cases are given. The study results can be widely used in engineering and also can be developed to the situation that passive sensors and targets aren't distributed in the same plane.

2. PROBLEM FORMULATIONS

2.1 Review of S-D Assignment Problem

In order to identify the number of targets and their position in field of detection, The likelihood function of \(Z_{i_1}, \ldots, i_s = \{Z_{i_1}, Z_{2i_2}, \ldots, Z_{si_s}\}\) originated from the same target is given as[3,4]

\[
\Lambda(Z_{i_1}, \ldots, i_s) = \prod_{s=1}^{S} [P_{ds} \cdot p(Z_{si_s})]^{1-\delta_{0s}}_s [1-P_{ds}]^\delta_{0s} (1)
\]

Where \(P_{ds}\) is the detect probability of sensor \(s\), \(\delta_{0s}\) is a binary indicator function, defined as:

\[
\delta_{0s} = \begin{cases} 
1 & \text{if } i_s = 0 \\
0 & \text{otherwise}
\end{cases}
\]

Where \(i_s = 0\) denotes that sensor \(s\) missed the detection, and \(p(Z_{si_s})\) denotes the probability density function of \(Z_{si_s}\) being from target \(t\). The likelihood that the measurements are all spurious or unrelated to target \(t\), i.e., \(t = \Phi\) is
\[ \Lambda(Z_{i_1\ldots i_S} | t = \Phi) = \prod_{s=1}^{S} \frac{1}{\Psi_s} \]  \hfill (3)

Where \( \Psi_s \) is the field of view of sensor \( s \), the cost of associating the 4-tuple of measurement to target \( t \) is given by

\[ c_{i_1\ldots i_s} = -\ln \left( \frac{\Lambda(Z_{i_1\ldots i_s} | t)}{\Lambda(Z_{i_1\ldots i_s} | t = \Phi)} \right) \]  \hfill (4)

Let \( \hat{\omega}_t \) denotes the position of \( t \) in (1), as \( \hat{\omega}_t \) is unknown, it is usually replaced by its maximum likelihood or least-squares estimation

\[ \hat{\omega}_t = \arg \max_{\omega_t} \Lambda(Z_{i_1\ldots i_s} | t) \]  \hfill (5)

Hence, the cost of associating \( Z_{i_1\ldots i_s} \) with target \( t \) is \[3,4]\]

\[ c_{i_1\ldots i_s} = \sum_{s=1}^{S} \left[ (1 - \delta_{\omega_t}) \ln(2\pi \cdot \delta_{\omega_t}) + \frac{1}{2} \frac{(Z_{si} - \hat{\Theta}_s \delta_{\omega_t})^2}{\sigma_s^2} \right] - \delta_{\omega_t} \cdot \ln(1 - P_{d_{i_s}}) \]  \hfill (6)

In order to find the most likely set of S-tuple of measurements such that each measurement is assigned to a target or declared false, and each measurement is assigned to at most one target, the data association problem can be transformed into the following generalized S-D assignment problem

\[ \min_{\rho_{i_1\ldots i_S}} \sum_{i_1 = 0}^{n_1} \sum_{i_2 = 0}^{n_2} \cdots \sum_{i_S = 0}^{n_S} c_{i_1\ldots i_s} \rho_{i_1\ldots i_S} \]  \hfill (7)

subject to

\[ \sum_{i_2 = 0}^{n_2} \sum_{i_3 = 0}^{n_3} \cdots \sum_{i_S = 0}^{n_S} \rho_{i_1\ldots i_S} = 1; \quad \forall i_1 = 1,2\ldots n_1 \]

\[ \sum_{i_2 = 0}^{n_2} \sum_{i_3 = 0}^{n_3} \cdots \sum_{i_S = 0}^{n_S} \rho_{i_1\ldots i_S} = 1; \quad \forall i_2 = 1,2\ldots n_2 \]

\[ \vdots \]

\[ \sum_{i_S = 0}^{n_S} \rho_{i_1\ldots i_S} = 1; \quad \forall i_S = 1,2\ldots n_S \]  \hfill (8)

2.2 Lagrangian Relaxation Algorithm

This S-D assignment problem mentioned above can be shown to be NP-hard\[3,4]\], the optimal techniques require unacceptable time and is of little practical value. Instead, fast and near the optimal solutions are most desirable. Among various heuristic algorithms, Lagrangian relaxation algorithm of multi-dimension assignment problem has a dominant role owing to its satisfying result in application. It relaxes the S-D assignment problem to a series of 2-D assignment problem to solve, which can be resolved by various algorithms in polynomial time. The advantage of this algorithm compared with the other modern optimal algorithm is that, when the algorithm is over, not only can be the suboptimal solution which is near to the optimal obtained, but also can be a measure of the quality of the solution obtained.

Because of the heavy burden from calculation of cost function of the generalized S-D assignment problem, the run time of Lagrangian relaxation algorithm is still longer, so we will present a modified algorithm which can save the procedure time in next section. The significance of the modified algorithm is that it uses a cross location method to eliminate some false intersection points, so the computation burden of calculating the cost function of assignment problem is decreased. The modified algorithms aiming at 3-D and 4-D assignment problem are shown respectively as follows.

3. THE MODIFIED ALGORITHM

![Fig.1. Position of 3 or 4 sensors](image1)

![Fig.2. Location of 3 or 4 sensors](image2)

Suppose the sensors’ position are shown as Fig.1, and three or four bearing-only sensors are used to locate targets as illustrated in Fig.2. Let \( \Theta_i \) \((i=1,2,3,4) \) denote the bearing measurements, the positions of sensors are denoted by \((x_i, y_i)\), \(i=1,2,3,4\), and the Cartesian position of \( A, B, C \) is denoted by \((x_A, y_A), (x_B, y_B)\) and \((x_C, y_C)\), hence\[5\]
\[
\begin{align*}
tg \theta_1 &= \frac{y_A - y_1}{x_A - x_1}, \quad tg \theta_2 = \frac{y_A - y_2}{x_A - x_2}, \\
tg \theta_3 &= \frac{y_B - y_3}{x_B - x_3}, \quad tg \theta_4 = \frac{y_C - y_3}{x_C - x_3}, \\
x_A &= \frac{y_2 - y_1 + x_1 tg \theta_1 - x_2 tg \theta_2}{tg \theta_1 - tg \theta_2}, \\
y_A &= \frac{y_2 tg \theta_1 - y_1 tg \theta_2 + (x_1 - x_2) tg \theta_1 tg \theta_2}{tg \theta_1 - tg \theta_2}
\end{align*}
\]

Let
\[
d_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}
\]  
(9)

Where, if \(x_A > x_B\), \(d_{AB}\) is positive. Otherwise, it is negative, \(d_{BC}\) and \(d_{AC}\) can be obtained in the same way. Let
\[
\begin{align*}
d_3 &= |d_{AB}| \\
d_4 &= |d_{AB}| + |d_{BC}| + |d_{AC}|
\end{align*}
\]

\(t_{AB} = d^2_{AB} / \sigma^2_{AB}\), \(t_{BC} = d^2_{BC} / \sigma^2_{BC}\)

\(t_{AC} = d^2_{AC} / \sigma^2_{AC}\)

Where \(\sigma_{AB}, \sigma_{BC}, \sigma_{AC}\) are variances of \(t_{AB}, t_{BC}\) and \(t_{AC}\) separately. They can be expressed approximately as:
\[
\begin{align*}
\sigma^2_{AB} &= (\partial d_{AB} / \partial \theta_1)^2 \sigma^2_{\theta_1} + (\partial d_{AB} / \partial \theta_2)^2 \sigma^2_{\theta_2}, \\
\sigma^2_{BC} &= (\partial d_{BC} / \partial \theta_3)^2 \sigma^2_{\theta_3} + (\partial d_{BC} / \partial \theta_4)^2 \sigma^2_{\theta_4}, \\
\sigma^2_{AC} &= (\partial d_{AC} / \partial \theta_5)^2 \sigma^2_{\theta_5} + (\partial d_{AC} / \partial \theta_6)^2 \sigma^2_{\theta_6}
\end{align*}
\]

Where \(\sigma_{\theta_1}, \sigma_{\theta_2}\) and \(\sigma_{\theta_3}\) is the standard measurement error of sensor 1, sensor 3 and sensor 4 respectively.

Construct the statistics:
\[
T_3 = t_{AB} = \frac{d^2_{AB}}{\sigma^2_{AB}}
\]  
(10)

\(T_4 = t_{AB} + t_{BC} + t_{AC} = \frac{d^2_{AB}}{\sigma^2_{AB}} + \frac{d^2_{BC}}{\sigma^2_{BC}} + \frac{d^2_{AC}}{\sigma^2_{AC}}\)  
(11)

\(T_3, T_4\) can be thought to follow distribution of \(\chi^2(1)\) and \(\chi^2(3)\) approximately. When a certain remarkable value \(\alpha\) is given, the approximate value \(d_3^*\) and \(d_4^*\) corresponding to the division point \(\alpha\) can be obtained by a number method. Let \(\tau_3 = d_3^*\), \(\tau_4 = d_4^*\). If the value of verdict function \(d_3\) or \(d_4\) for an arbitrarily 3-tuple or 4-tuple of measurements has one of the following conclusion: \(d_3 < \tau_3\) or \(d_4 < \tau_4\) correspondingly, then the 3-tuple or 4-tuple of measurements is originated from a same target probably, receive it. Otherwise, it can be removed from the candidate association.

In reality, the Cartesian position of A, B and C don’t coincide because of the measurement error. The probability that the measurements come from a same target can be estimated by the value of \(d_3\) or \(d_4\). Obviously, the smaller the size is, the more possible that they come from the same target has. We use the value of \(d_3\) or \(d_4\) to be the verdict function to determine if we should calculate the cost of 3-tuple or 4-tuple of measurements. In order to avoid the disadvantages of heavy burden of calculation to solve the nonlinear equation to obtain the gating and the influence of inferior condition that statistics may be not obey to the \(\chi^2\) distribution equation strictly, according to the large number of simulations with the various measurement errors, it can be determined by \(\tau_3 = 300 \times 180 / \pi \times \sigma\) and \(\tau_4 = 500 \times 180 / \pi \times \sigma\) respectively. Where \(\sigma\) denotes the bearing measurement error of each sensor, if it is different from various sensors, \(\sigma\) can be replaced by the average of three or four values. This mainly because that the gating of verdict function \(d_3\) and \(d_4\) changes with the bearing measurement error, but is not sensitive with the separation of sensors and targets.

As the association cost of S-D assignment problem is defined as (4), so the corresponding S-tuple \(Z_{i_1...i_s}\) may be considered a candidate association if and only if \(t_{i_1...i_s} < \tau\), and all the S-tuples \(Z_{i_1...i_s}\) with \(c_{i_1...i_s} > 0\) can be eliminated from the list of candidate associations by a certain measure. The sparsity of candidate association is defined as the ratio of the number of potential measurement-target association in the S-D assignment problem to the number of a fully connection. In the modification algorithm, we use the sparsity of candidate association is a little lower than the former, so the false location points with lower cost value can be removed from the cost matrix, hence, the accuracy of data association can be improved properly. For the purpose of distinguishing the sparsity after modification with the former one, we denote the former sparsity as \(s_0\), and the later one is \(s_\tau\). Where \(s_\tau\) is a function of the gating value \(\tau\) for verdict function, which is just be taken as an approximate gating using experimental function \(\tau_3\) or \(\tau_4\).

In reality, we can also adjust the size of the threshold value by observing the number of cost function in the pages of two-dimension at different circumstance at any time adaptively. In the case of S-D assignment problem, the sparsity of candidate target is usually lower. According to the accuracy of direction-finding cross location in current, the ratio of the number of cost function need to be calculated with the total number of cost function is about 5-15% in the pages of two-dimension. From the above-mentioned discussion, only those S-tuple of measurements, which have passed through the gating of verdict function and the corresponding values of cost function are negative, can be permitted to enter the optimal assignment process, i.e., the accurate association process, and the process which makes use of the verdict function to remove the false location points can be regarded as a rough association process.
4. PERFORMANCE OF SIMULATION

4.1 Simulation Model

The sensors’ position are shown in Fig.1, suppose the position of four sensors are (-2000, 0), (-1000, -1000 $\sqrt{3}$), (1000, -1000 $\sqrt{3}$) and (2000, 0), and the position of three sensors are (-2000, 0), (0, -2000) and (2000, 0). Targets are distributed in lines y=800km and y=1000lm equidistantly. The bearing measurement errors of various sensor are the same value, we use them, $\sigma_\theta$ of 0.3°, 0.7°. The detection probability of each sensor is assumed to be 0.95, and the false alarm rate is assumed 0. In the simulation of this paper, we adopt the method of using the experimental function to decide taking or rejecting the calculation of cost function, and a certain detection measure is supposed to be adopted to avoid the influence of the earth curvature.

4.2 Analysis of Simulation Results

As shown in table 1 and 2, with the modification, the run times are both reduced in two various cases. This is because that a large quantity of calculation from the false location points is cut. The association accuracy corresponding to $s_r < s_0$ is advantage of the result of $s_r = s_0$ in which $s_0$ is the sparsity corresponding to the total negative cost element. The main reason of above result is some false location points are removed from the candidate association with the limitation of the gating of verdict function, and this causes that the accuracy of data association and location are improved. In the case of $s_r = s_0$, the accuracy of data association and location are almost in accordance with the results before modification, however the run time decreased largely. Compare the results of table1 and table2, it is known that the accuracy of data association and location is improved and the run time also increased greatly with the increase of dimension of assignment problem. It is also known that the advantage for saving time of the modified algorithm aiming at 4-D assignment problem is more obvious than the corresponding result of 3-D assignment problem. This result demonstrates that the higher the dimension of the assignment problem has, the more obvious the superiority of the modified algorithm is.

Table1: Average result (25 runs) of 3-D assignment algorithm in different measurement errors (number of targets =20)

<table>
<thead>
<tr>
<th>$0.3^0$</th>
<th>association accuracy</th>
<th>Run (s)</th>
<th>time (s)</th>
<th>RMS error (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>former</td>
<td>68.8%</td>
<td>8.65</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>Late ($s_r \approx s_0$)</td>
<td>69.1%</td>
<td>2.75</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>Later ($s_r &lt; s_0$)</td>
<td>75.8%</td>
<td>2.67</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>$0.7^0$</td>
<td>former</td>
<td>60.5%</td>
<td>9.45</td>
<td>2.27</td>
</tr>
<tr>
<td>Late ($s_r \approx s_0$)</td>
<td>61.0%</td>
<td>3.12</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>Later ($s_r &lt; s_0$)</td>
<td>66.2%</td>
<td>2.93</td>
<td>1.72</td>
<td></td>
</tr>
</tbody>
</table>

Table2: Average result (25 runs) of 4-D assignment algorithm in different measurement errors (number of targets =20)

<table>
<thead>
<tr>
<th>$0.3^0$</th>
<th>association accuracy</th>
<th>Run (s)</th>
<th>time (s)</th>
<th>RMS error (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>former</td>
<td>85.5%</td>
<td>72.35</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>Late ($s_r \approx s_0$)</td>
<td>85.1%</td>
<td>27.38</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>Later ($s_r &lt; s_0$)</td>
<td>93.2%</td>
<td>24.02</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>$0.7^0$</td>
<td>former</td>
<td>77.5%</td>
<td>79.35</td>
<td>1.93</td>
</tr>
<tr>
<td>Late ($s_r \approx s_0$)</td>
<td>77.8%</td>
<td>31.24</td>
<td>1.97</td>
<td></td>
</tr>
<tr>
<td>Later ($s_r &lt; s_0$)</td>
<td>84.9%</td>
<td>27.96</td>
<td>1.42</td>
<td></td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

This text mainly studies the application of Lagarangian relaxation algorithm of S-D assignment problem. A modified algorithm is presented based on statistics. The threshold of the verdict function is given. The new algorithm reduces the run time of the global optimal algorithm of S-D assignment problem, and improves the effect of data association and location. This algorithm can be developed to situation that the sensors and targets are not in the same plane, and has widely applied foreground in reality.

REFERENCE


BIOGRAPHIES

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