A very fast direct torque control for interior permanent magnet synchronous motors start up

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Abstract

Selection of the stator flux command in direct torque control (DTC) of permanent magnet synchronous motors is very important as it affects machine performance. For steady state operation, it can help to decrease the stator current or machine losses. However, in order to get a fast torque response during motor start up, one needs to change both the stator flux angle and magnitude. In this paper, a method is presented to find the optimal voltage vectors to change both the flux angle and its magnitude that results in high speed dynamics for the torque response. The closed form formula given for the optimal voltage vector is derived based on maximizing the torque change in each sampling time. The simulation results show that the presented method is superior to the conventional DTC. In other words, it is shown that the voltage vectors derived from the switching table for conventional DTC are not the optimal voltage vectors for the situation in which there is a step change in the torque command.

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1. Introduction

Direct torque control (DTC) is considered as one of the best alternatives for motor drive designers in order to get a fast torque response; especially when torque control instead of speed or position control, is the control objective. Besides high torque dynamics, it is well known for being robust to motor parameters change, except stator resistance [1]. For the first time, its application in a permanent magnet synchronous (PMS) motor was discussed in Ref. [2]. The authors also...
investigated the effect of the flux reference on the torque dynamics, finding that a bigger flux reference results in a bigger achievable torque but a slower torque response. They also found a maximum value for the flux reference, but they did not present any strategy to select the flux command during torque transitions, i.e. when there is a step change in torque command like motor start up.

The flux command is usually kept constant and equal to the nominal value for speeds under the nominal speed. However, this does not give the maximum torque to amp ratio. For speeds over the nominal speed, flux weakening is applied to the vector control of an IPM (interior permanent magnet) synchronous motor [3]. An algorithm is also proposed to select the flux command for two different objectives: minimization of the stator current and minimization of the total motor losses [4]. The proposed algorithm is proper for steady state operation and still does not provide any improvement to torque dynamics because it determines a constant flux linkage. A qualitative description is also presented to achieve a fast torque response in PMS motors [5]. However, no mathematical derivation or system synthesis is attempted.

In the current paper, a method, independent of the flux command, is proposed to get a fast torque response for an interior permanent magnet (IPM) motor start up from standstill. The proposed method looks for an optimal voltage vector that results in a maximum torque change in each sampling time.

In Section 2, the model of an IPM machine, besides the fundamentals of DTC, is presented. To show the relationship between the flux command and torque dynamics, the effect of choosing different flux commands for the same machine is investigated in Section 3. In Section 4, a new strategy is presented to calculate the optimal voltage vector in order to get rapid torque dynamics. The presented method is evaluated by simulating an IPM motor, and the results are compared with ones obtained by traditional DTC. Finally, Section 5 summarizes the main contributions of this paper.

2. IPM machine model and DTC fundamentals

Using the $d$–$q$ transformation, the voltage equations of an IPM machine in the rotor reference frame are as follows:

\[ v_d = R_s i_d + \frac{d \lambda_d}{dt} - \omega_c \lambda_q, \]  

\[ v_q = R_s i_q + \frac{d \lambda_q}{dt} + \omega_c \lambda_d, \]  

where $\lambda_d = L_d i_d + \lambda_m$, $\lambda_q = L_q i_q$ and the stator flux linkage is $\lambda_s = (\lambda_d + \lambda_q)^{\frac{1}{2}}$.

The corresponding equivalent circuits are shown in Fig. 1.

It has been shown that the electromagnetic torque in an IPM machine can be regulated by controlling the magnitude and angle of the stator flux linkage or load angle $\delta$ as seen in Fig. 2 [2]. This can be performed by applying the proper output voltage vectors of an inverter to the machine. There are six nonzero voltage vectors and one zero voltage vector for a two level inverter as depicted in Fig. 3. They can be represented by:
Fig. 1. Equivalent circuit model.

Fig. 2. Stator flux linkage vector diagram.

Fig. 3. Voltage vectors of a two level inverter.
The authors in Ref. [2] have discussed the relationship between the amplitude of the stator flux linkage command and the derivative of the electromagnetic torque (or torque dynamics). They also showed that in order to achieve a fast torque response, the flux command should be selected properly during the torque transient. However, no attempt has been made to determine the method of flux selection. In this section, the influence of the flux linkage amplitude and angle on the motor torque is further analyzed to provide a basis for the optimal voltage selection presented.
in the next section. Fig. 4 shows the electromagnetic torque with respect to \( \delta \) for four different flux commands for an IPM machine whose specification is given in Appendix A.

It is obvious that the smaller values of the flux command result in linear and fast torque responses with respect to \( \delta \). However, the maximum achievable electromagnetic torque is smaller with the smaller values of the flux. This is a disadvantage, especially in the case when the machine is heavily loaded and needs bigger electromagnetic torque to start up rapidly. So, it is important to vary the stator flux properly during torque transition to overcome the mentioned shortcoming. In the next section, an effective strategy will be proposed to find an optimum voltage vector for varying both the stator flux angle and amplitude in order to obtain a maximum torque increase in each sampling period.

4. Optimal voltage vector

4.1. Optimal voltage vector selection

The torque of Eq. (5) can also be represented in the rotor reference frame as:

\[
T_e = \frac{3}{2} P \left[ \lambda_m + (L_d - L_q)i_d \right] i_q. \tag{6}
\]

Then, the torque differential with respect to \( i_d \) and \( i_q \) is obtained as:

\[
\mathrm{d}T_e = \frac{3}{2} P \left\{ (L_d - L_q)i_q \mathrm{d}i_d + [\lambda_m + (L_d - L_q)i_d] \mathrm{d}i_q \right\}. \tag{7}
\]
During each sampling period $T_s$, Eq. (7) can be approximated by:

$$\Delta T_e = A \Delta i_d + B \Delta i_q,$$

where

$$A = \frac{3}{2} P (L_d - L_q) i_q,$$

$$B = \frac{3}{2} P [\lambda_m + (L_d - L_q) i_d].$$

By neglecting the voltage drop across the stator resistance, the following equations are easily obtained from the machine voltage Eqs. (1) and (2) along the $d$ and $q$ axes:

$$\Delta i_d = \frac{v_d + E_d}{L_d} T_s, \quad E_d = \omega_e \lambda_q,$$

$$\Delta i_q = \frac{v_q - E_d}{L_q} T_s, \quad E_q = \omega_e \lambda_d.$$

Substituting Eqs. (11) and (12) into Eq. (8) yields:

$$\Delta T_e = f(v_d, v_q) = av_d + bv_q + c,$$

where

$$a = \frac{A T_s}{L_d}, \quad b = \frac{B T_s}{L_q} \quad \text{and} \quad c = \left( \frac{A E_d}{L_d} - \frac{B E_q}{L_q} \right) T_s.$$

A very fast torque response requires that $\Delta T_e$ has a maximum value all the time. This is possible by applying an optimal voltage vector to the machine in each sampling period. This voltage is found here in terms of its $d$ and $q$ components where

$$v_q = \sqrt{\left( \frac{2}{3} V_{DC} \right)^2 - v_d^2}.$$ 

Substituting Eq. (15) into Eq. (13), the optimal voltage is the solution of the following equation

$$\frac{\partial f(v_d)}{\partial v_d} = 0,$$

which, in connection with Eq. (15), yields:

$$v_d' = \frac{2a V_{DC}}{3\sqrt{a^2 + b^2}},$$

$$v_q' = \frac{2b V_{DC}}{3\sqrt{a^2 + b^2}}.$$
Eqs. (17) and (18) represent the \(d\) and \(q\) components of the optimal voltage vector \(V^*_s\) which has been derived based on maximizing the torque change in each sampling time \(T_s\). In the next section, space vector modulation (SVM) is used to apply the calculated optimum voltage to IPM motors.

### 4.2. SVM to apply optimum voltage vector to machine

The optimal voltage vector was obtained in the rotor reference frame above. However, DTC is basically performed in a stationary reference frame. In this section, a transformation from the rotor reference frame to the stationary reference frame is proposed with a minimal requirement to change the rotor angle. Then, space vector modulation (SVM) is used to apply the optimum voltage vector to the motor.

By using Eqs. (17) and (18) and the initial rotor position \(\theta_r\), the optimal voltage vector components in the stationary reference frame are found as follows:

\[
\begin{bmatrix}
    v^*_D \\
    v^*_Q
\end{bmatrix} = \begin{bmatrix}
    \cos \theta_r & -\sin \theta_r \\
    \sin \theta_r & \cos \theta_r
\end{bmatrix} \begin{bmatrix}
    v^*_d \\
    v^*_q
\end{bmatrix}.
\] (19)

During the torque development duration, which typically takes shorter than 10 ms, the rotor position \(\theta_r\) does not change too much and can be assumed constant. Therefore, by knowing the initial rotor position, an encoder is not needed. In practice, once the coefficient matrix in Eq. (19) is calculated, it can be stored in memory and used in each sampling time. Thus, unlike in vector control, here the \(dq\) to \(DQ\) transformation does not take much time for a processor. Now, SVM, which is basically a pulse width modulation (PWM) control technique that produces minimum harmonic distortion, is used to represent the calculated optimum voltage in terms of the output voltage vectors of a two level inverter. In this method, by using superposition, one can write:

\[
V^*_s T_s = V_{kT_a} + V_{k+1T_b}
\] (20)

in which \(V_k\) and \(V_{k+1}\) are two voltage vectors that are adjacent to \(V^*_s\) and are 60\(^\circ\) out of phase. The time duration of applying these voltage vectors, \(t_a\) and \(t_b\) respectively, are obtained by solving Eq. (20). The results are represented by:

\[
t_a = \frac{3T_s}{2V_{DC}} \left[ v^*_D - \frac{v^*_Q}{\sqrt{3}} \right],
\] (21)

\[
t_b = \frac{\sqrt{3}T_s}{V_{DC}} v^*_Q.
\] (22)

Since the sum of the values calculated for \(t_a\) and \(t_b\) should be equal to the sampling time \(T_s\), they should be normalized as:

\[
t_{an} = \frac{t_a T_s}{t_a + t_b},
\] (23)

\[
t_{bn} = T_s - t_{an}.
\] (24)
5. System performance

5.1. System block diagram

The block diagram of the proposed DTC is depicted in Fig. 5. Based on the equations derived for the optimal voltage vector, Eqs. (17) and (18), instead of using a constant flux command, as used for conventional DTC, a new block, i.e. optimal voltage calculation, and a software switch “S” has been added to the conventional DTC system. The proposed DTC has two modes of operation:

- During starting, in which there is a step change in the torque command, the switch S is ‘1’ and the optimal voltage in each sampling time is calculated by using Eqs. (17) and (18) and applied to the machine through the inverter.
- When the machine torque reaches its set point value, i.e. it is between the hysteresis bands, the switch S is ‘0’ and the hysteresis band regulators are used to maintain the flux and torque control as in a conventional DTC system.

5.2. Simulation results

The system performance under the proposed DTC strategy is evaluated, and the results are compared with ones obtained under the conventional DTC with constant flux command. In both cases, an IPM motor with the data in Appendix A is tested. The motor is supposed to start up from standstill with the set point equal to 200% nominal torque (nominal torque is 3.96 Nm). The minimum flux linkage command that should be used for conventional DTC for this motor is $1.3\lambda_m$ as seen in Fig. 4. Choosing a flux linkage command less than $1.3\lambda_m$ leads the control system to be unstable because the maximum achievable machine torque will be less than the torque command.

It is seen in Fig. 6 that a faster torque response is achieved by the proposed DTC. The conventional DTC tends to increase both the stator flux and torque during starting while the motor...
draws positive $i_d$ and $i_q$ as seen in Figs. 7 and 8 respectively. Since, in IPM machines, the direst axis inductance $L_d$ is smaller than the quadrature axis inductance $L_q$, a positive $i_d$ produces a

Fig. 6. Machine torque $T_e$ during motor start up.

Fig. 7. Stator direct axis current $i_d$ during motor start up.
Fig. 8. Stator quadrature axis current $i_q$ during motor start.

Fig. 9. Stator flux locus and phase current $I_a$. (a) Conventional DTC with $\lambda^* = 1.3 \lambda_m$ and (b) proposed method.
negative reluctance torque according to Eq. (6). In contrast, the proposed DTC tends to increase the flux angle rather than its amplitude, which can be easily seen in the flux trajectory shown in Fig. 9. The motor also absorbs a negative $i_d$ that produces a positive reluctance torque, which results in a fast torque response.

Fig. 10 shows the rotor angle and its electrical speed. It is clearly seen that the rotor position has changed less than 0.5° during the torque development duration, which is almost 3 ms. This supports the assumption we made before that the rotor position is almost constant during the torque development duration and just the initial position is needed for the reference transformation.

6. Conclusions

In order to get a fast torque response, a new strategy and a closed form formula are presented to find the optimal voltage vector during motor start up from standstill. The obtained formula is based on maximizing the torque change in each sampling time. In contrast to the traditional DTC, the presented strategy changes both the stator flux angle and amplitude, which results in a linear and fast torque response.

For further work, one can devise a new switching table using the idea introduced in this paper. The new switching table can help us get high torque dynamics without increasing the computations incurred to calculate the optimal voltage vector.
Acknowledgements

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Appendix A. Motor data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated speed, rpm</td>
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</tr>
<tr>
<td>$L_d$, mH</td>
<td>42.44</td>
</tr>
<tr>
<td>$L_q$, mH</td>
<td>79.57</td>
</tr>
<tr>
<td>Rated torque, N m</td>
<td>3.96</td>
</tr>
<tr>
<td>Rated current, A</td>
<td>3</td>
</tr>
<tr>
<td>$\lambda_m$, Wb</td>
<td>0.314</td>
</tr>
<tr>
<td>$P$, no. of pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>$R_s$, $\Omega$</td>
<td>1.93</td>
</tr>
<tr>
<td>$J$, inertia N m s$^2$/rad</td>
<td>0.003</td>
</tr>
<tr>
<td>$B$, viscous friction</td>
<td>0.008</td>
</tr>
</tbody>
</table>

References