A Ray Tracing Simulator Based on 3GPP TR 25.996 v.6.1.0
Spatial Channel Model for Multiple Input Multiple Output (MIMO) Simulations
A Ray Tracing Simulator Based on 3GPP TR 25.996 v. 6.1.0

User’s Guide
Version 1.0

Spatial Channel Model for MIMO Simulations
A Ray Tracing Simulator Based on 3GPP TR 25.996 v. 6.1.0

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History

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1 Description

This document suggests a user’s guide to a MATLAB application which simulates a Spatial Channel Model (SCM) based on reference [1] with some minor adjustments. It focuses on how to use the Graphical User Interfaces (GUIs) and the functions of the application and avoids going into specifics about the nature of the parameters as this is done analytically in the reference document. The user should at any point consult [1] for additional information.

The MIMO spatial channel model simulates a wireless propagation channel in various cases and applies the concept of diversity (spatial and polarization) assuming multiple antennas at both the transmitter and receiver, thus forming a Multiple Input Multiple Output antenna system.

2 Scope

The scope of the present SCM application is to provide an easy-to-use MATLAB developed application to any user who requires a practical tool to perform MIMO simulations and to obtain statistical data which can be later further analyzed. Much effort was made so that all the parameters involved are user-selective, something that allows this application to be flexible to any desired case.
3 Glossary, Assumptions and Simplifications

3.1 Glossary

[SISO] Single Input Single Output
[MIMO] Multiple Input Multiple Output
[IR] Impulse Response
[SNR] Signal to Noise Ratio
[PDF] Probability distribution function
[CDF] Cumulative distribution function
[LOS] Line of Sight
[NLOS] No Line of Sight
[BS] Base Station
[MS] Mobile Station
[SF] Shadow Fading
[PL] Path Loss
[PDP] Power Delay Profile
[DS] Delay Spread
[AoA] Angle of Arrival
[AoD] Angle of Departure
[AS] Angle (Azimuth) Spread
[PAS] Power Azimuth Spectrum
[DoT] Direction of Travel
[DL] Downlink
[UL] Uplink
[LN] Log-Normal
[RV] Random Variable
3.2 Assumptions and Simplifications

In this section we note some general assumptions and simplifications made by [1] and adopted by the application. Further assumptions will be made for each case scenario but those will be noted in each corresponding section.

1. Ray Tracing Simulator

This application uses a ray-based method, where the reception at a given point is composed by the sum of all rays (paths) arriving to the antenna at every instant after being scattered by objects of the area. Each ray is described by its power and delay and can be decomposed to a large number of sub-rays (subpaths) which we assume to be plane electromagnetic waves who share a common frequency (the carrier frequency of the transmitted signal), each one arriving with a random phase. The sub-rays that belong to the same ray have common powers and delays.

2. Limitation to 2 Dimensions

This simulator assumes that all waves propagate parallel to the x-y plane i.e. it neglects the elevation spectrum. This is a common assumption made when we refer to outdoor scenarios as in those cases the z-axis wave components are not significant. Contrarily, this assumption cannot be made when we study indoor scenarios where a 3-D model is required. Indoor scenarios are not included in this application but should be included in future versions.

3. Uplink-Downlink Reciprocity

Since there is no indication of alternate channel behavior during the uplink and the downlink, the AoD/AoA values are identical between both propagating directions, although this does not apply to the random subpath phases during the UL and DL which will be assumed to be uncorrelated. In other words, we assume that the channel is a double directional system where the BS and MS can both be the receiver and transmitter despite the fact that we will refer to the BS as the transmitter.

4. Single Base Station-Single Mobile Station link

The application simulates a (NLOS) link between a single transmitter (Base Station) who is placed in the center of a hexagonal cell and a single receiver (Mobile Station) who can move within the same cell. Cases of additional BSs or MSs are not included i.e. shadowing effects from other antennas are not examined.

5. Linear Arrays (space diversity)

The concept of space diversity is applied by using antenna arrays at both the BS and the MS. Here, only linear arrays are simulated. Every element of each array is described by the same antenna pattern which is selected by the user. We define the arrays broadside to be a vertical line to the line connecting the array elements.

6. Polarized Arrays (polarization diversity)
Since hand-handled devices have limited dimensions, space diversity might be difficult to be applied to
them. For that reason, polarized arrays which use cross shaped, co-located dipole antenna pairs might be
the primary way to apply (polarization) diversity. This application also simulates this type of diversity.

7. Line of Sight

In this version, only NLOS cases are considered. LOS cases will be included in future versions.

8. Noise

Noise is neglected.

4 Spatial Channel Model for Simulations

4.1 Environments and Cases

During the BS/MS link there are a large number of phenomena that take place and affect the signal reception
like the Path Loss, the Shadow Fading, the Fast Fading, the Doppler shift etc. For that reason, there are many
parameters and variables that should be taken under consideration during a simulation in order for it to
produce reliable results for each scenario. Let us now present a rough description of the environments and
cases examined in this application. More details about them can be found in [1].

- Environments

We study three different environments; the Suburban Macrocell, the Urban Macrocell and the Urban
Microcell. The first two have statistical similarities and they follow the same modeling process with some
parameter adjustment so we can categorize them both as Macrocell. In general, the code follows the same
simulation steps for all three environments shown in the reference document [1]. For this reason, we will limit
our presentation to the areas where the application handles some parameters differently. However, let us note
some basic characteristics for each environment.

- Macrocell –
  - Approximately 3 km distance BS to BS
  - BS antenna above rooftop height
  - The adopted pathloss model is the modified COST231 Hata urban propagation model
  - Angle spread, Delay spread and Shadow fading will be treated as correlated, LN random variables

- Microcell –
  - Less than 1 km distance BS to BS
  - BS antenna is at rooftop height
  - The adopted pathloss model is the COST 231 Walfish-Ikegami NLOS model
Angle spread, Delay spread and Shadow fading will not be treated as correlated RV

- **Cases**

The sub-rays leaving the BS can change their polarization before reaching the MS. For example, while the BS transmits only vertical polarized sub-rays, the MS can receive vertical, horizontal and off-axis polarized sub-rays. The Cross Polarization Discrimination (XPD) describes the intensity of this phenomenon and it is defined as the ratio of the co-polarized average received power ($P_{co-pol}$) to the cross-polarized average received power ($P_{cross-pol}$). Note that since: $P_{co-pol} \geq P_{cross-pol}$, then $XPD \geq 0$.

- **Case I** –
  - It neglects the cross polarization
  - BS and MS antennas transmit and receive only vertical polarized sub-rays
  - BS antennas will be assumed directional (sector) or omnidirectional antennas (vertical polarized ideal dipoles), while the MS antennas will be assumed omnidirectional (vertical polarized ideal dipoles).

- **Case II** –
  - It includes the cross polarization
  - BS and MS antenna can transmit and receive both vertical and horizontal polarized sub-rays
  - BS and MS array elements will be assumed (tilted) ideal dipoles or (tilted) cross-polarized, co-located dipoles, forming dipole pairs. Note that when dipole pairs are used, the number of the antennas at each array is twice the number of the array elements.

Cases I and II are handled by different functions and some of their features are not the same. For example, the first case assumes three kinds of antennas for the BS; hence, the signal’s attenuation will also be dependent from the antenna’s gain function.

We should note that since Case I is less complicated, it produces faster simulations and for that reason it should be used when there is interest for space diversity results or results concerning the usage of directional antennas. Case II should be used for polarization diversity or mixed (spatial and polarization) diversity results. More will be said about the diversification of the two cases since we will examine them separately when needed.

### 4.2 Channel realization

At this point, let us define ‘drop’ as a single simulation run where a BS using an array of $S$ elements transmits inside a terrestrial environment to a moving MS using an array of $U$ elements for a given number of time frames. The signal arrives to the receiver through $N$ independent paths which are described by their powers and delays. In this way we form $N$, time evolving, $SxU$ matrixes and their sum would describe the total channel realization:
Our goal is to generate the coefficients \( h_{s,u,n}(t) \) \((s=1...S, u=1...U)\) for every \( H_{S,U,n}(t) \) \((n=1...N)\) for every time frame.

The equation that describes these time dependant coefficients is common for each environment but it alters when we refer to cases I and II. Next, we will present these equations for each case giving the description for every participating parameter. Some of these parameters are handled as Input parameters selected by the user while others derive from RV and are generated inside the application's code. Each parameter will be analyzed in chapter 6.

### Case I

The \((s, u)\) matrix component for \( H_{S,U,n}(t) \) for all three environments will be:

\[
h_{u,s,n}(t) = \sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \left[ \sqrt{G_{BS}(\theta_{n,m,AoD})} \exp[j(kd_s \sin(\theta_{n,m,AoD}) + \Phi_{n,m})] \times \sqrt{G_{MS}(\theta_{n,m,AoA})} \exp[j(kd_u \sin(\theta_{n,m,AoA})] \times \exp( jk v \cos(\theta_{n,m,AoA} - \theta_n) t) \right] \]


\( P_n \) is the power of the \( n \)th path.

\( N \) is the number of paths (clusters).

\( M \) is the number of subpaths per-path.

\( S \) is the number of the BS linear array antenna elements.

\( U \) is the number of the MS linear array antenna elements.

\( \Phi_{n,m} \) is the phase of the \( m \)th subpath of the \( n \)th path.

\( \theta_{n,m,AoD} \) is the AoD for the \( m \)th subpath of the \( n \)th path.

\( \theta_{n,m,AoA} \) is the AoA for the \( m \)th subpath of the \( n \)th path.

\( G_{BS}(\cdot) \) is the BS antenna gain of each array element.

\( G_{MS}(\cdot) \) is the MS antenna gain of each array element for.
j is the square root of -1.

k is the wave number \(2\pi / \lambda\) where \(\lambda\) is the carrier wavelength in meters.

\(d_s\) is the distance in meters from BS antenna element \(s\) from the reference \((s = 1)\) antenna. For the reference antenna \(s = 1\), \(d_1 = 0\).

\(d_u\) is the distance in meters from MS antenna element \(u\) from the reference \((u = 1)\) antenna. For the reference antenna \(u = 1\), \(d_1 = 0\).

\(v\) is the magnitude of the MS velocity vector.

\(\theta_s\) is the angle of the MS velocity vector.

\[\text{Case II}\]

The \((s, u)\) matrix component for \(H_{S,U,n}(t)\) for all three environments will be:

\[
h_{u,s,n}^{(H)}(t) = \frac{r_2}{\sqrt{M}} \sum_{m=1}^{M} \begin{bmatrix} G^{(v)}(\theta_{n,m, AoA})^T \exp(j\Phi_{n,m}^{(v,v)}) \sqrt{r_1} \exp(j\Phi_{n,m}^{(v,h)}) \\ G^{(h)}(\theta_{n,m, AoA}) \sqrt{r_2} \exp(j\Phi_{n,m}^{(h,v)}) \exp(j\Phi_{n,m}^{(h,h)}) \end{bmatrix} \times \exp(jkd_s \sin(\theta_{n,m, AoD})) \exp(jkd_u \sin(\theta_{n,m, AoA})) \exp(jkv \cos(\theta_{n,m, AoA} - \theta_v) t)
\]

\(G_{BS}^{(x)}(.)\) is the BS antenna gain of each array element for the x direction (either horizontal h or vertical v).

\(G_{MS}^{(x)}(.)\) is the MS antenna gain of each array element for the x direction (either horizontal h or vertical v).

\(r_1\) is the power ratio of waves of the \(n\)th path leaving the BS in the vertical direction and arriving at the MS in the horizontal direction (v-h) to those leaving in the vertical direction and arriving in the vertical direction (v-v).

\(r_2\) is the power ratio of waves of the \(n\)th path leaving the BS in the horizontal direction and arriving at the MS in the vertical direction (h-v) to those leaving in the vertical direction and arriving in the vertical direction (v-v). We assume that: \(r_1 = r_2\).

\(\Phi_{n,m}^{(x,y)}\) is the phase of the \(m\)th subpath of the \(n\)th path between the x component (either the horizontal h or vertical v) of the BS element and the y component (either the horizontal h or vertical v) of the MS element. Note that all the other parameters inside eq. (4) are the same as Case I.

To help us with the spatial description of the BS, the MS and all the angles we introduce the following angle parameters (common for all environments and cases).

\(\Omega_{BS}\) is the BS antenna array orientation, defined as the difference between the broadside of the BS array and the absolute North (N) reference direction.
\( \theta_{BS} \) is the LOS AoD direction between the BS and MS, with respect to the broadside of the BS array.

\( \delta_{n, AoD} \) is the AoD for the \( n \)th (\( n = 1 \ldots N \)) path with respect to the LOS AoD.

\( \Delta_{n,m,AoD} \) is the offset for the \( m \)th (\( m = 1 \ldots M \)) subpath of the \( n \)th path with respect to \( \delta_{n, AoD} \).

\( \Omega_{MS} \) is the MS antenna array orientation, defined as the difference between the broadside of the MS array and the absolute North (N) reference direction.

\( \theta_{MS} \) is the angle between the BS-MS LOS and the MS broadside.

\( \delta_{n, AoA} \) is the AoA for the \( n \)th (\( n = 1 \ldots N \)) path with respect to the LOS AoA.

\( \Delta_{n,m,AoA} \) is the offset for the \( m \)th (\( m = 1 \ldots M \)) subpath of the \( n \)th path with respect to \( \delta_{n, AoA} \).

Note that (see Fig. 1):

\[
\theta_{n,m,AoD} = \theta_{BS} + \delta_{n, AoD} + \Delta_{n,m,AoD}
\]

\[
\theta_{n,m,AoA} = \theta_{MS} + \delta_{n, AoA} + \Delta_{n,m,AoA}
\]

Finally, we note that clockwise angles are considered positive and anti-clockwise are considered negative. Concerning angles \( \Omega_{BS}, \Omega_{MS}, \theta_{BS}, \theta_{MS} \) for simplicity, instead of negative values we are going to use their explementary positive values. For example, if \( \theta_{MS} = -15^0 \) we will assume that: \( \theta_{MS} = 360^0 - 15^0 = 345^0 \). This way we only handle positive angles. However, this does not apply to angles \( \delta_{n, AoD}, \delta_{n, AoA}, \Delta_{n,m,AoD} \) and \( \Delta_{n,m,AoA} \) which can also take negative values.

Then:

\[
\theta_{MS} = |\Omega_{BS} - \Omega_{MS} + \theta_{BS} + 180^0|
\]
5 Installation

This application was developed using MATLAB 7.1.0.246 (R14) Service Pack 3 August 02, 2005 though older versions might also support it.

Installation and Running:

1) Extract all ‘SCM.zip’ file content to a folder (e.g. C:\Matlab\work\SCM).
2) Change MATLAB’s ‘Current Directory’ to the directory used above for extraction.
3) Type ‘SCM’ to MATLAB’s Command Window to run the application.

6 Options and Parameters

In this section we will analyze the simulator’s options and parameters. The user can get some first information about them when using the program by clicking on the graphic interface of the application on the option or parameter of interest. The application will pop-up a help dialog window, as shown below, giving some basic information about the parameter or option and providing the corresponding pages in the manual.

6.1 Options

The application’s options are subcategorized to Initial and Menu options.
6.1.1 Initial Options

![Initial Options Window]

*Fig. 2* The application initializes by displaying this first window where the Initial Options are shown.

Environment

Following the instructions found in [1] the model simulates three different environments:

- the Suburban Macrocell
- the Urban Macrocell
- the Urban Microcell

![Environment Options]

*Fig. 3* Choose which environment to simulate

Polarization

This option determines whether the simulations will include only vertical polarized sub-paths or both vertical and horizontal polarization. If the user chooses Case I: ‘Only Vertical’ then the BS and MS antennas will receive and transmit only vertical propagating sub-rays and the XPD will not affect the simulation. If the user chooses Case II: ‘Vertical and Horizontal’ then the BS and MS can receive and transmit both vertical and horizontal polarized sub-paths.
Enable Plotting

Here the user toggles plotting ‘on’ and ‘off’ i.e., whether there’s going to be a graphical display of various parameter plots like the signal’s fast fading, the total channel capacity, the MS temporal autocorrelation and the power-delay profile after the completion of each simulation run. Plotting also includes a graphic display of the cell, the scatters, the BS and the MS and the AoDs/AoAs.

After the user is done with the Initial Options he should press the ‘Continue’ button placed at the bottom right corner of the window shown in Fig. 2. The application will proceed to the next window where all the rest of the Options and Input Parameters can be found.
6.1.2 Menu Options

Antenna Patterns

CASE I

- Base Station Antenna Pattern

There are three options for the BS antenna; the 3 sector, the 6 sector and the omnidirectional antenna (vertical polarized ideal dipole). Real cellular systems usually use directional antennas at the BS (sector antennas). Each element of the BS array will be described by the same selected antenna pattern and the signal attenuation will be a given by:

\[ A_{BS}(\theta) = -\min \left[ 12 \left( \frac{\theta}{\theta_{3dB}} \right)^2, A_m \right] \]

\[ -180^\circ < \theta < 180^\circ \]

\( \theta \) is defined as the angle between the direction of interest and the boresight (the direction of the maximum gain) of the antenna, \( \theta_{3dB} \) is the 3dB beamwidth in degrees and \( A_m \) the maximum attenuation. For the 3 sector antenna, \( A_m = 20 \text{ dB} \) and \( \theta_{3dB} = 70^\circ \) while for the 6 sector antenna \( A_m = 23 \text{ dB} \) and \( \theta_{3dB} = 35^\circ \).

![Choose the antenna pattern for every BS antenna array element.](image)

- Mobile Station Antenna Pattern

Each element of the MS array will be assumed as an ideal, vertical polarized dipole with respect to the x-y plane (omnidirectional antenna), hence:

\[ A_{MS}(\theta) = 0 \text{ dB}. \]

For all antennas, the antenna numeric gain function is given by:

\[ G(\theta) = 10^{A(\theta)/10} \]

(5)
Fig. 6  The antenna pattern for the 3 sector cell.

Fig. 7  The directions of the arrows show the boresights of the 3 antennas that transmit inside a 3 sector cell.
Fig. 8  The antenna pattern for the 6 sector cell.

Fig. 9  The directions of the arrows show the boresights of the 6 antennas that transmit inside a 6 sector cell.

**Note:** Since we assume only one BS and one MS inside a cell, we place the BS in the center of the cell with its boresight facing the absolute North. In other words, the boresight and the broadside of the BS linear array are identified. This applies to both cases I and II. More will be said about this when we discuss the Orientation Options.
CASE II

In Case II we have both vertical and horizontal polarized sub-paths propagating inside the channel, hence, it would be more practical to assume arrays of ideal dipoles at both the BS and MS. These dipoles can be tilted will respect to the z-axis by a common polarization angle $a_{BS}$ and $b_{MS}$ respectively. The antenna gain for the vertical and horizontal reception will be given by the matrix:

$$
\begin{bmatrix}
G^{(v)}(\theta) \\
G^{(h)}(\theta)
\end{bmatrix} = \begin{bmatrix}
\cos \alpha \\
\sin \alpha \cos \theta
\end{bmatrix}
$$

$\theta$ is the angle that the sub-ray arrives/departs to/from the dipole and $\alpha$ is the polarization angle.

This case was developed to perform spatial and polarization diversity simulations and it gives the user the ability to choose from two different array elements for both the BS and MS separately. The first choice assumes single (tilted) dipoles which form a linear array while the second assumes that every array element is formed by two (tilted) cross polarized co-located dipoles (dipole pairs). In this way, there are four different combinations of BS and MS arrays.

![Fig. 10 Choose between a dipole array and a cross-polarized co-located dipole pair array for both the BS and MS](image)

Note that two dipoles, $i$ and $i+1$ in a dipole pair, have the same spacing from the reference element of the array hence: $d_i = d_{i+1}$ and for their polarization angles: $|\alpha_i - \alpha_{i+1}| = 90^\circ$.

**Note:** To go from ‘Primary’ to ‘Secondary’ Options and Parameters click ‘Additional Properties’.

**Assign Path Power and Delays**

While the reference document calculates the power and the delay of each path through parameters that are randomly distributed, this application can provide the freedom to manually input the power and the delay of each path. In other words, setting this option to ‘on’, the user can form the PDP according to his own likings. If this option is set to ‘off’, then those parameters will be calculated though the step procedure of [1]. The number of $\{\tau_n, P_n\}$ pairs in the PDP is dependant from the value of $N$ which gives the number of paths (clusters). These input values have an effect to some other parameters which derive from random procedures (see Table 2).
Switch this option 'on' to manually input the Power and the Delay for each Path forming the Power Delay Profile.

Two Power Delays Profiles, the first one using the default power-delay values and the second one formed by the random procedure of the code.
Orientation Options

This option dictates whether the orientation of the MS (i.e. its distance from the BS and all its angle parameters) will be random (through an automatic random procedure provided by the application) or custom (where the user inputs all the orientation parameters manually). All of the orientation parameters will be explained analytically when we discuss the channel's parameters.

![Orientation Options](image1)

However, when ‘Random’ is chosen we place the Base Station array at a fixed location with its broadside and boresight facing the Absolute North as shown in Fig. 14. This can be changed when ‘Custom’ is chosen though angle $\Omega_{BS}$ (see orientation parameters).

![Fig. 14 The BSs fixed location inside the cell when ‘Random’ option is chosen](image2)
Plot Options

At the bottom right part of the ‘Additional Properties’ are the ‘Plot Properties’ where the user chooses which drop, path and link between the \textit{DxNxsxU} links to plot after the simulation is over.

![Fig. 15 Choose which link to use for plotting](image)

### 6.2 Channel Parameters

The channel’s parameters are subcategorized to primary and secondary. Secondary parameters can be found when clicking ‘Additional Properties’.

#### Primary Input Parameters

![Fig. 16 Primary Input Parameters for Case I](image)
Let us now explain each parameter, giving instructions where needed.

- **$S$** Number of the BS linear antenna array elements.

- **$U$** Number of the MS linear antenna array elements.

- **$d_{BS}$** Distance between neighboring elements at the BS linear array in wavelengths. Input a 1x ($S$-1) matrix where its element $s$ gives the spacing in wavelengths between array elements $s$ and $s+1$ ($s=1...S-1$).

- **$d_{MS}$** Distance between neighboring elements at the MS linear array in wavelengths. Input a 1x ($U$-1) matrix where its element $u$ gives the spacing in wavelengths between array elements $u$ and $u+1$ ($u=1...U-1$).

- **$BSAS$** Base Station per path Angle Spread in degrees. $BSAS$ defines the statistics of the BS sub-paths AoD through RV $\Delta_{AoD}$ (see Table 2).

- **$MSAS$** Mobile Station per path Angle Spread in degrees. $MSAS$ defines the statistics of the MS sub-paths AoA through RV $\Delta_{AoA}$ (see Table 2).

- **$N$** Number of Paths (clusters or rays).
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- \( M \) Number of sub-paths (sub-rays).

- \( f_c \) Carrier frequency in GHz.
  The carrier frequency is associated with the wave number \( k \) through: \( k = 2\pi f_c/c \) where \( c \) is the propagating velocity (\( c = 3 \times 10^8 \) m/sec).

- \( v \) Mobile Station velocity vector magnitude in km/h.
  The velocity of the MS will affect the intensity of the Doppler shift (the maximum Doppler shift is given by: \( f_{d,max} = \pm v/\lambda \) where \( \lambda \) is the carrier wavelength). For pedestrian cases \( v \) should be around 3 km/h while for vehicular cases it should be around 30 km/h (slow vehicular) or 120 km/h (fast vehicular).

- \( t \) Time duration of ‘drop’ in seconds.
  Define the simulation’s duration.

- \( T \) Time frame of ‘drop’ in milliseconds .
  Define the sampling period. The number of simulation instants will be: \( t = \text{floor}(t/T) \).

- \( D \) Number of ‘drops’.
  The application supports multi-simulation runs defined by \( D \). These simulations will have all the input parameters common while all the parameters that are randomly distributed will be re-generated in each ‘drop’.

- \( XPD \) Cross Polarization Discrimination in dB (Case II only).
  In [1], the \( XPD \) (defined in chapter 4) is handled as a RV. Here we find it plausible to consider it as an input parameter.

- \( a_{BS} \) Base Station antenna(s) tilt with respect to the z-axis in degrees (Case II only).
  Define the antenna tilt at the BS array. Note that this tilt will be common for every antenna at the BS array.

- \( \beta_{MS} \) Mobile Station antenna(s) tilt with respect to the z-axis in degrees (Case II only).
  Define the antenna tilt at the MS array. Note that this tilt will be common for every antenna at the MS array.

Additional/Secondary Input Parameters

- \( R \) Cellular hexagon radius in meters.
  If \( d \) is the BS to BS distance in meters, then \( R = d/\sqrt{3} \).

- \( SNR \) Signal to Noise Ratio in dB
SNR is needed to calculate the channel’s capacity.

**Suburban and Urban Macrocell secondary input parameters**

- $r_{DS}$ \( \sigma_{\text{delay}}/\sigma_{DS} \)
  - $r_{DS}$ defines the statistics of the path delays through RV $\tau$ (see Table 2).

- $r_{AS}$ \( \sigma_{\text{AoD}}/\sigma_{AS} \)
  - $r_{AS}$ defines the statistics of the BS paths AoD through RV $\delta_{\text{AoD}}$ (see Table 2).

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<td>Cellular hexagon radius in meters</td>
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</tr>
<tr>
<td>Signal To Noise Ratio in dBs</td>
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<td>15</td>
</tr>
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</table>

**Urban Microcell secondary input parameters**

- $DS$ \( \text{Delay Spread in microseconds} \)
  - $DS$ defines the statistics of the path delays through RV $\tau$ (see Table 2).

- $BSppD$ \( \text{BS per-path AoD Distribution} \)
  - $BSppD$ defines the statistics of the BS paths AoD through RV $\delta_{\text{AoD}}$ (see Table 2).

<table>
<thead>
<tr>
<th>Secondary Input Parameters</th>
<th>$DS$</th>
<th>$BSppD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay Spread in microseconds</td>
<td>1.2</td>
<td>40</td>
</tr>
<tr>
<td>Cellular hexagon radius in meters</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Signal To Noise Ratio in dBs</td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

**Orientation Parameters**

- $d$ \( \text{Distance between the BS and the MS in meters} \)
- $\Omega_{BS}$ \( \text{BS antenna array orientation, defined as the difference between the broadside of the BS array and the absolute North (N) reference direction} \)
- $\Omega_{MS}$ \( \text{MS antenna array orientation, defined as the difference between the broadside of the MS array and the absolute North (N) reference direction} \)
- $\Theta_{BS}$ \( \text{LOS AoD direction between the BS and MS, with respect to the broadside of the BS array} \)
- Theta_MS Angle between the BS-MS LOS and the MS broadside.
- Thetav Angle of the velocity vector, with respect to the broadside of the MS array.

**Correlated Parameters σ_AS, σ_DS, σ_SF**

- \( r_{DS-AS} \) Correlation between Delay Spread and Angle Spread.
- \( r_{SF-DS} \) Correlation between Shadow Fading and Delay Spread.
- \( r_{SF-AS} \) Correlation between Shadow Fading and Angle Spread.
- \( \sigma_{SH} \) Shadow Fading standard deviation in dB.
- \( e_{DS} \) Delay Spread logarithmic standard deviation, \( e_{DS} = e_{DS} = \sqrt{E[\log_{10}(\sigma_{DS})]} - \mu_{DS} \)
- \( m_{DS} \) Delay Spread logarithmic mean, \( m_{DS} = m_{DS} = E(\log_{10}(\sigma_{DS})) \)
- \( e_{AS} \) Angle Spread logarithmic standard deviation, \( e_{AS} = e_{AS} = \sqrt{E[\log_{10}(\sigma_{AS})]} - \mu_{AS} \)
- \( m_{AS} \) Angle Spread logarithmic mean, \( m_{AS} = m_{AS} = E(\log_{10}(\sigma_{AS})) \)
Fig. 18 Define here the correlation properties for the correlated RV delay spread, angle spread and shadow fading. Notice that when the Urban Microcell environment is chosen, all Correlated Parameter properties are disabled except $\sigma_{SH}$.

In Table 1 we concentrate all the primary, secondary and additional parameters, their symbols, their default or suggested value for each environment and case and their numerical type or numerical limitations. Here, we should comment that the user must be careful when inputting the parameter values because some of them have a direct effect on the simulation duration (parameters $N$, $M$, $D$, $S$, $U$, $t$, $T$). We confine ourselves to limit only parameters $N$ and $M$ (to 18 and 200 respectively).
<table>
<thead>
<tr>
<th>Option/Parameter</th>
<th>Symbol</th>
<th>Default/Suggested Value</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment Option</td>
<td>ENV</td>
<td>Suburban Macrocell</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of BS linear array elements</td>
<td>S</td>
<td>2</td>
<td>Natural</td>
</tr>
<tr>
<td>Number of MS linear array elements</td>
<td>U</td>
<td>2</td>
<td>Natural</td>
</tr>
<tr>
<td>BS array elements spacing</td>
<td>d_{BS}</td>
<td>6 \lambda, 4 \lambda, 2 \lambda</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>MS array elements spacing</td>
<td>d_{MS}</td>
<td>0.4 \lambda</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Per path AS at BS</td>
<td>BSAS</td>
<td>2^0, 2^0, 5^0</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Per path AS at MS</td>
<td>MSAS</td>
<td>35^0</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Number of paths</td>
<td>N</td>
<td>6</td>
<td>Natural</td>
</tr>
<tr>
<td>Number of subpaths</td>
<td>M</td>
<td>20</td>
<td>Natural</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>f_c</td>
<td>2 GHz</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>MS velocity magnitude</td>
<td>v</td>
<td>60 km/h</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Time duration of drop</td>
<td>t</td>
<td>0.1 s</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Time frame of drop</td>
<td>T</td>
<td>1 ms</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Number of drops</td>
<td>D</td>
<td>2</td>
<td>Natural</td>
</tr>
<tr>
<td>Cross Polarization Discrimination</td>
<td>XPD</td>
<td>15 dB, 10 dB, 8 dB</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>BS array antenna(s) tilt</td>
<td>\alpha_{BS}</td>
<td>0^0</td>
<td>[0^0, 90^0]</td>
</tr>
<tr>
<td>MS array antenna(s) tilt</td>
<td>\beta_{MS}</td>
<td>0^0</td>
<td>[0^0, 90^0]</td>
</tr>
<tr>
<td>Hexagonal Radius</td>
<td>R</td>
<td>1700, 1200, 500</td>
<td>Pos. Real</td>
</tr>
<tr>
<td>Signal To Noise Ratio</td>
<td>SNR</td>
<td>15 dB</td>
<td>Real</td>
</tr>
<tr>
<td>\sigma_{AoD}/\sigma_{AS} ratio</td>
<td>r_{AS}</td>
<td>1.2, 1.3</td>
<td>N/A, Pos. Real</td>
</tr>
<tr>
<td>\sigma_{delay}/\sigma_{DS} ratio</td>
<td>r_{DS}</td>
<td>1.4, 1.7</td>
<td>N/A, Pos. Real</td>
</tr>
<tr>
<td>BS per-path AoD Distr.</td>
<td>BSppD</td>
<td>N/A</td>
<td>40^0, [0^0, 180^0]</td>
</tr>
<tr>
<td>Delay Spread</td>
<td>DS</td>
<td>N/A</td>
<td>1.2 \mu s, Pos. Real</td>
</tr>
<tr>
<td>LN Shadowing std dev</td>
<td>\sigma_{S\lambda}</td>
<td>8 dB, 8 dB, 10 dB</td>
<td>Real</td>
</tr>
<tr>
<td>Angle Spread at BS</td>
<td>\sigma_{AS}</td>
<td>\mu_{AS}=0.69, \epsilon_{AS}=0.13</td>
<td>Real</td>
</tr>
<tr>
<td>Delay Spread</td>
<td>\sigma_{DS}</td>
<td>\mu_{DS}=-6.80, \epsilon_{DS}=0.288</td>
<td>N/A</td>
</tr>
<tr>
<td>AS-DS Correlation</td>
<td>\rho_{AS-DS}</td>
<td>0.6, 0.6</td>
<td>Real</td>
</tr>
<tr>
<td>SF-AS Correlation</td>
<td>\rho_{SF-AS}</td>
<td>-0.5, -0.5</td>
<td>Real</td>
</tr>
<tr>
<td>SF-DS Correlation</td>
<td>\rho_{SF-DS}</td>
<td>-0.5, -0.5</td>
<td>Real</td>
</tr>
</tbody>
</table>

Table 1  Primary and Secondary channel parameters and their default/suggested values for each Environment and Case.

Note: Parameters BSAS and MSAS are the per-path AS at the BS and MS respectively. If we combine all path angles, they should result to the mean AS at the BS and MS, $E(\sigma_{AS,BS})$, $E(\sigma_{AS,MS})$. The user should be careful that the combination of $N$ and the BS and MS angle spread values produces the corresponding mean AS. For example, if we set $N=1$, we should set $BSAS = E(\sigma_{AS,BS})$, $MSAS = E(\sigma_{AS,MS})$. For Table 1, the corresponding BS and MS mean angle spreads are shown in the table below. For more information see: [1], p. 17, Table 5.1.
Random Variables

As mentioned before, there are parameters generated inside the code through a number of random variables, each one following its own distribution. In Table 2 we note these variables and the statistics that company them. It is obvious that some distributions depend on some Input parameters which we also include in Table 2. Since details about the origin of these distributions can be found in [1], we will limit our report on them by just presenting Table 2. Note that angles \( \Delta_{AoA} \), \( \Delta_{AoD} \) are RV and not fixed values like in [1].

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Suburban Macrocell</th>
<th>Urban Macrocell</th>
<th>Urban Microcell</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>~( r_{DS} \sigma_{DS} \ln z ), ( z \sim N(0,1) )</td>
<td>( U(0, \sigma_{DS}), \sigma_{DS}^2 = DS )</td>
<td>( ) )</td>
</tr>
<tr>
<td>( P )</td>
<td>( \exp((1-r_{DS})/r_{DS} \sigma_{DS}) \times 10^{\alpha} \times (\zeta/10), )</td>
<td>( 10^{\beta} \times (\tau+z/10), z \sim N(0,3dB) )</td>
<td>( ) )</td>
</tr>
<tr>
<td>( \theta_{AoA} )</td>
<td>( N(0, \sigma_{AoA}^2) ), ( \sigma_{AoA} = 104.12 \times (1-\exp(-0.2175</td>
<td>10\log(P_{n})</td>
<td>)) )</td>
</tr>
<tr>
<td>( \theta_{AoD} )</td>
<td>( N(0, \sigma_{AoD}^2) ), ( \sigma_{AoD} = r_{AS} \sigma_{AS} )</td>
<td>( U(-b,+b), ) ( b \equiv BSppD )</td>
<td>( ) )</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>( U(0^0, 360^0) )</td>
<td>( ) )</td>
<td>( ) )</td>
</tr>
<tr>
<td>( \Omega_{AS} )</td>
<td>( \theta_{MS} =</td>
<td>\Omega_{BS} + \Omega_{MS} + \theta_{BS} + 180^0</td>
<td>)</td>
</tr>
<tr>
<td>( \theta_{BS} )</td>
<td>( \sigma_{AS} = 10^{\gamma} \times (\epsilon_{AS}^2 + \mu_{AS}) )</td>
<td>( ) )</td>
<td>( ) )</td>
</tr>
<tr>
<td>( \theta_{MS} )</td>
<td>( \sigma_{DS} = 10^{\gamma} \times (\epsilon_{DS}^2 + \mu_{DS}) )</td>
<td>( ) )</td>
<td>( ) )</td>
</tr>
<tr>
<td>( \sigma_{SF} )</td>
<td>( \sigma_{SF} = 10^{\gamma} \times (\epsilon_{SF}^2 + \mu_{SF}) )</td>
<td>( ) )</td>
<td>( ) )</td>
</tr>
</tbody>
</table>

Table 2 Summary of the RV and their distributions

Since experimental measurements indicate that \( \sigma_{SF} \), \( \sigma_{AS} \) and \( \sigma_{DS} \) follow a LN distribution and, in addition, are correlated to each other, a model should reproduce these correlations. Thus, RV \( \alpha \), \( \beta \) and \( \gamma \) of Table 2 are correlated Gaussian RV and they derive from the procedure presented in [1], Chapter 5.6. Since we assume only one site, we set \( \zeta = 0 \), hence matrix B = 0. In other words the procedure simplifies to the one given in [2], p.530. However, this does not apply to urban micro environment where the correlation of \( \sigma_{SF} \), \( \sigma_{AS} \) and \( \sigma_{DS} \) is not reproduced (see Table 2).
7 Functions

In this section we describe the code’s functions. We will be very brief since MATLAB allows access to the functions’ code where someone can observe more details and even make alterations on it.

Multipath and MultipathPol

Functions ‘Multipath’ and ‘MultipathPol’ are the main functions of the application. ‘Multipath’ handles Case I while ‘MultipathPol’ handles Case II. When executed they can process all the environments, they generate the angles $\delta_{AoA,n}, \delta_{AoD,n}$ and the $SxU$ matrix $H_{S,U,n}$ for every $n (n=1…N)$ and for every instant. They also call functions ‘DIST’, ‘CorParameters’, ‘FastFading’ or ‘FastFadingPol’ respectively, ‘PDPmacro’ or ‘PDPmicro’ (depending on the chosen environment) and ‘OrientationsFixBS’.

FastFading and FastFadingPol

‘FastFading’ and ‘FastFadingPol’ are called by functions ‘Multipath’ and ‘MultipathPol’ and handle cases I and II respectively. They generate angles: $\Phi_{n,m}$ or $\Phi^{(x,y)}_{n,m}$, $\Delta_{AoA,n,m}$, $\Delta_{AoD,n,m}$ and the summations (3) or (4) respectively, for every instant which represents the channel’s fast fading. ‘FastFading’ can also call functions ‘PDPmacro’ or ‘PDPmicro’ when a sector antenna is selected in Case I.

PDPmacro and PDPmicro

Functions ‘PDPmacro’ and ‘PDPmicro’ provide the Power Delay Profile for the macrocell and microcell environment respectively. They generate parameters $P_n$ and $\tau_n$ following the directions given in [1] (also see Table 2).

CorParameters

Function ‘CorParameters’ generates $\sigma_{AS}$, $\sigma_{DS}$, $\sigma_{SF}$ following the procedure in [1] or [2] and reproduces the correlation between angle spread, delay spread and shadow fading. It is called by function ‘Multipath’ or ‘MultipathPol’ as $\sigma_{AS}$ is used to generate angles $\delta_{AoD,n}$ and $\sigma_{DS}$ is used to generate delays $\tau_n$ (see Table 2) while $\sigma_{SF}$ is the output parameter for shadow fading. ‘CorParameters’ is not called when the user has selected the Urban Microcell environment.
OrientationsFixBS

If ‘Random’ is chosen at the ‘Orientation Parameters’, function ‘OrientationFixBS’ gives random values to: \(d, \Omega_{MS}, \theta_{BS}, \theta\), while it sets \(\Omega_{BS} = 0\) (fixed boresight and broadside direction, see Fig. 14) and calculates:

\[
\theta_{MS} = |\Omega_{BS} - \Omega_{MS} + \theta_{BS} + 180^\circ|
\]

G3, G6

Functions ‘G3’ and ‘G6’ determine the antenna attenuation and give the numeric gain \(G(\theta)\) for the 3-sector and the 6-sector directional antenna respectively. In other words they reproduce the numeric gain of the antenna patterns of Fig. 6 and Fig. 8. They are called inside function ‘FastFading’ when the BS is calibrated to use a 3-sector or a 6-sector antenna.

DIST

Function ‘DIST’ calculates the spacing in meters between the reference antenna element of an array (the first one in the linear array) and each other elements of the same array.

Capacity

Function ‘Capacity’ calculates the channel’s total capacity \(C(t)\) for every time instant \(t\), through the equations shown below. Equation (7) is the classic equation used to estimate the capacity in a MIMO antenna system.

\[
C(t) = \sum_{n=1}^{N} C_n(t) \quad (6)
\]

\[
C_n(t) = \log_2 \left[ \det \left( \begin{bmatrix} \text{SNR} & \text{H}_{S,U,n}(t) \end{bmatrix} \begin{bmatrix} \text{H}_{S,U,n}(t) & \text{I}_U \end{bmatrix} \right) \right] \quad (7)
\]

where: \(N\) is the number of paths, \(S\) and \(U\) are the number of the antenna elements at the transmitter and the receiver respectively, \(\text{SNR}\) is the signal to noise ratio, \(\text{I}_U\) is the \(U \times U\) unit matrix, \(\det\) indicates the determinant, \(\text{H}_{U,S,n}\) is given in equation (2) and \(\text{H}^\dagger_{S,U,n}\) is its inverse complex conjugate matrix.
8 Output Parameters

In this chapter we are going to discuss the Output Parameters of the application. Firstly, every output of each simulation can be found inside a data{i}.mat file which is exported to a folder chosen from the user, where i represents the number of simulation (try not to confuse ‘simulation’ with ‘drop’ since a simulation may include a number of ‘drops’ that depends on Input parameter D). The default path is: C:\Matlab\work but the user can change this path by clicking the ‘Save to…’ button and choosing another folder to save the file. These files (.mat) can be opened with MATLAB and further processed by the user.

Each data{i}.mat contains ten output parameters shown in Table 3. These outputs are in a form of arrays either Double or Cell. Double is an array of double-precision numbers while Cell is an array of indexed cells, each capable of storing an array of a different dimension and data type. In Table 3 one can observe each parameter’s name, symbol, type, dimension and unit.

Note: Output Parameters AS and DS are not generated when we study the urban microcell environment.
Spatial Channel Model for MIMO Simulations
A Ray Tracing Simulator Based on 3GPP TR 25.996 v.6.1.0

User’s Guide v. 1.0

<table>
<thead>
<tr>
<th>Output Parameter</th>
<th>Symbol</th>
<th>Array Type</th>
<th>Dimensions</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel coefficients, ( h_{s,u,n}(t) )</td>
<td>( H )</td>
<td>Cell</td>
<td>{1xD}{Nx1}{tx1}{SxU}</td>
<td>10^*(dB/10)</td>
</tr>
<tr>
<td>Path Angle of Arrival, ( \delta_{n,AoA} )</td>
<td>( AoA )</td>
<td>Cell</td>
<td>{1xD}{Nx1}</td>
<td>Degrees</td>
</tr>
<tr>
<td>Path Angle of Departure, ( \delta_{n,AoD} )</td>
<td>( AoD )</td>
<td>Cell</td>
<td>{1xD}{Nx1}</td>
<td>Degrees</td>
</tr>
<tr>
<td>Path Power, ( P_n )</td>
<td>( P )</td>
<td>Cell</td>
<td>{1xD}{Nx1}</td>
<td>10^*(dB/10)</td>
</tr>
<tr>
<td>Path Delay, ( \tau_n )</td>
<td>( t )</td>
<td>Cell</td>
<td>{1xD}{Nx1}</td>
<td>(\mu s)</td>
</tr>
<tr>
<td>Angle Spread, ( \sigma_{AS} )</td>
<td>( AS )</td>
<td>Double</td>
<td>1xD</td>
<td>Degrees</td>
</tr>
<tr>
<td>Delay Spread, ( \sigma_{DS} )</td>
<td>( DS )</td>
<td>Double</td>
<td>1xD</td>
<td>(\mu s)</td>
</tr>
<tr>
<td>Shadow Fade, ( \sigma_{SF} )</td>
<td>( SF )</td>
<td>Double</td>
<td>1xD</td>
<td>10^*(dB/10)</td>
</tr>
<tr>
<td>Path Loss, ( PL{dB} )</td>
<td>( PL )</td>
<td>Double</td>
<td>1xD</td>
<td>dB</td>
</tr>
<tr>
<td>Channel Capacity, ( C(t) )</td>
<td>( C )</td>
<td>Cell</td>
<td>{1xD}{tx1}</td>
<td>bps/Hz</td>
</tr>
</tbody>
</table>

Table 3  Properties of the Output parameters stored inside each data{i}.mat file

For example, if we set \( D = 2 \), \( N = 6 \), \( t = 0.1s \), \( T = 1 \text{ ms} \) (thus we have \( t = t/T = 100 \) time instants), \( S = 2 \) and \( U = 3 \), the \( H \) output parameter will be an \{1x2\}{6x1}{100x1}{2x3} cell array as shown in Fig. 20.

![Array Editor - H{1,2}{2,1}{10,1}](image)

**Fig. 20**  The \(SxU\) complex channel coefficients for the 2nd ‘drop’, the 2nd path and the 10th time instant

We should also comment that .mat files can also be handled outside MATLAB for users using C or other platforms. The way to do that can be found analytically inside MATLAB Help.
9 Plots

In this final chapter we will present a number of plots that derive from the usage of the application’s functions. These plots have two purposes; firstly to test the correctness of the code by plotting a number of correlations and comparing them with the theoretical correlation curves. Secondly, we can estimate the channel’s capacity and view its dependence on a number of parameters and make conclusions about the usage of MIMO antennas as a way to increase the wireless propagation channel’s capacity.

9.1 Correlations

For correlations we will be using the normalized correlation coefficient:

\[
 r(x) = \text{Re} \left\{ \int_{-\infty}^{+\infty} h(\chi) h^*(\chi - x) d\chi \right\} \int_{-\infty}^{+\infty} h^2(\chi) d\chi
\]

\(\chi\) can represent time, distance or angle depending on the type of correlation and \(x\) is its relative shift.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D)</td>
<td>20</td>
<td>(M)</td>
<td>20</td>
</tr>
<tr>
<td>(\theta_{BS})</td>
<td>0°</td>
<td>(\theta_{MS})</td>
<td>0°</td>
</tr>
<tr>
<td>SNR</td>
<td>15 dB</td>
<td>(f_C)</td>
<td>2 GHz</td>
</tr>
<tr>
<td>(\theta_e)</td>
<td>0°</td>
<td>(v)</td>
<td>30 km/h</td>
</tr>
<tr>
<td>(\theta_{AOD})</td>
<td>(N(0, \sigma_{AS,BS}^2))</td>
<td>(\sigma_{AS,BS})</td>
<td>8°</td>
</tr>
<tr>
<td>(\theta_{AoA})</td>
<td>(U(0, 2\pi))</td>
<td>(XPD)</td>
<td>0 dB</td>
</tr>
<tr>
<td>Case I</td>
<td>Omni (Vpol dipoles)</td>
<td>Case II</td>
<td>Tilted dipoles</td>
</tr>
</tbody>
</table>

Table 4 Values for Fig. 20 to 24.

Let us begin by estimating the spatial cross correlation between two closely separated antennas at both the MS and the BS, as a function of their spacing, \(d\). We will examine both cases I and II and for the simulations we use the parameter values of Table 4. Note, that in Table 4 we only have one path (\(N=1\)) and for that reason, we use the mean AS at the BS and not the per-path AS. In addition, we assume that \(\theta_{AoA} \sim U(0, 2\pi)\). These adjustments are necessary because most bibliography assumes only one path and unity distribution for the AoA.

We will study the correlation at the MS first. The MS spatial correlation functions for eq. (3) and (4) will respectively be:

\[
 r^{(I)}(d) = \frac{1}{M} \sum_{m=1}^{M} \exp(jkd \sin \theta_{n,m,AoA})
\]
where: \( \alpha, \alpha' \) are the tilts for each antenna (case II). Since we assume \( M \) to be a large number, we can replace the sums with integrals, and because \( \theta_{AoA} \sim U(0, 2\pi) \) we will get:

\[
r^{(II)}(d) = \frac{1}{M} \left[ \cos(\alpha - \alpha') \sum_{m=1}^{M} \exp(jkd \sin \theta_{n,m,AoA}) - \sin(\alpha) \sin(\alpha') \sum_{m=1}^{M} \exp(jkd \sin \theta_{n,m,AoA}) \sin^2(\theta_{n,m,AoA}) \right]
\]

(9)

where: \( J_0(\cdot) \) is the first kind, zero order Bessel function and \( J_2(\cdot) \) is the first kind, second order Bessel function.

Note that in eq. (11), if \( \alpha = \alpha' = 0^\circ \) (vertical polarized dipoles), we go back to \( J_0(kd) \).

In Fig. 20 we can view the MS’s spatial correlations \( r^{(I)} \) and \( r^{(II)} \) for cases I and II respectively. We see that in case I (i.e. vertical polarized dipoles) the correlation of eq. (1) is very close to \( J_0(kd) \), where \( r(d) \) reaches zero for the first time after \( 0.38 \lambda \). In case II we use the extreme case where \( \alpha = 90^\circ \) and \( XPD = 0 \text{ dB} \) \( (r = 1) \). We observe that the simulation is now close to \( J_0(kd) \)-\( J_2(kd) \) and reaches zero approximately at \( 0.31 \lambda \). It is obvious that for all other dipole polarizations the correlation would reach zero somewhere between 0.31 and 0.38 \( \lambda \).
When we study the BS, the correlations are the same as eq. (8) and (9) if we replace angles $\theta_{n,m,AoD}$ with $\theta_{n,m,AoD}$, but we cannot jump to eq. (10) and (11) because $\theta_{AoD} \sim N(0, \sigma_{AS,BS}^2)$. Nevertheless, there is a theoretical curve that estimates this type of correlations and stands for small angle spreads and is given by [3]:

$$ f(d) = \exp\left[jkd \sin(\bar{\theta}_{AoD})\right] \exp\left[-\frac{1}{2} (kd\sigma_{AS,BS} \cos \bar{\theta}_{AoD})^2\right] = \exp\left[-\frac{1}{2} (kd\sigma_{AS,BS})^2\right] $$

(12)

because: $\bar{\theta}_{AoD} \equiv \theta_{BS} = 0$

For the BS we study four different angle spreads as shown in Fig. 22. We can see that the simulation plots are very close to $f(d)$. These curves derive using the values of Table 4 for Case I.

Let’s go back to eq. (11); for co-located dipoles ($d = 0$) the correlation between them is a function of their relative antenna tilt $\alpha - \alpha'$. We can estimate this angular correlation by keeping one dipole vertical polarized and tilting the second one, hence:

$$ r^{(II)}(a) = \cos(a) $$

(13)

The correlation result which is shown if Fig. 23 is in perfect agreement with eq. (13)
Next, we will discuss the MS’s temporal correlation. In Table 4 we assume $\theta_v = 0$, so the temporal correlation for each case will be:

$$r^{(1)}(t) = \frac{1}{M} \sum_{m=1}^{M} \exp(jkt \cos \theta_{n,m,\alpha})$$

$$r^{(1)}(t) = \frac{1}{M} \left[ \sum_{m=1}^{M} \exp(jkt \cos \theta_{n,m,\alpha}) - \sin^2(\alpha) \sum_{m=1}^{M} \exp(jkt \cos \theta_{n,m,\alpha}) \sin^2(\theta_{n,m,\alpha}) \right]$$

Here, $\alpha = \alpha'$ because we have the same dipole moving (autocorrelation). Hence:

$$r^{(1)}(t) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \exp(jkt \cos \theta_{A0A}) \, d\theta_{A0A} = f_0(kvt)$$

$$r^{(1)}(t) = \frac{1}{2\pi} \left[ \int_{-\pi}^{+\pi} \exp(jkt \cos \theta_{A0A}) d\theta_{A0A} - \sin^2(\alpha) \int_{-\pi}^{+\pi} \exp(jkt \cos \theta_{A0A}) \sin^2(\theta_{A0A}) d\theta_{A0A} \right]$$

$$= f_0(kvt) - \sin^2(\alpha) f_2(kvt)$$

We would have reached the same conclusion if we set $d = vt$ in eq. (10) and (11). In other words, the MS’s temporal correlation and spatial cross correlation are identified when: $\alpha = \alpha'$, something that is also confirmed by Fig. 24.
The following plots concern the RV $\sigma_{AS}$, $\sigma_{DS}$, $\sigma_{SF}$ which are correlated with each other. Remember that this correlation is taken under consideration for macrocell environments and not microcell. For the plots, we use the properties of suburban macrocell environment shown in Table 5. In Fig. 25 we can observe the positive correlation between $\sigma_{AS}$ and $\sigma_{DS}$, while in Fig. 26 and 27 we notice the negative correlation of $\sigma_{SF}$ with $\sigma_{DS}$ and $\sigma_{AS}$ respectively.

![Mobile Station Temporal Correlation](image)

**Fig. 24  Temporal Correlation at the MS for case I and II**

<table>
<thead>
<tr>
<th>LN Shadowing std dev</th>
<th>$\sigma_{SH}$</th>
<th>$\sigma_{AS}$</th>
<th>$\sigma_{DS}$</th>
<th>$\sigma_{SF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Spread at BS</td>
<td>$\mu_{AS}$=0.69 \ $\sigma_{AS}$=0.13 \ $E(\sigma_{AS})$=5.0</td>
<td>$\mu_{DS}$=-6.80 \ $\sigma_{DS}$=0.288 \ $E(\sigma_{DS})$=0.17 $\mu_s$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delay Spread</td>
<td>$\rho_{AS-DS}$</td>
<td>$\rho_{SF-AS}$</td>
<td>$\rho_{SF-DS}$</td>
<td></td>
</tr>
<tr>
<td>AS-DS Correlation</td>
<td>0.6</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>SF-AS Correlation</td>
<td>-0.5</td>
<td>-0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SF-DS Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 5  Properties for $\sigma_{AS}$, $\sigma_{DS}$, $\sigma_{SF}$ for the Suburban macrocell environment.**
Fig. 25  Delay spread versus Angle spread for 1000 ‘drops’.

Fig. 26  Shadow fading versus Delay spread for 1000 ‘drops’.
Finally, it would be plausible to show that these correlated parameters actually follow a log normal distribution. This can be done by plotting their CDFs. We will do this for parameters $\sigma_{AS}$, $\sigma_{DS}$ for the suburban and urban macrocell environment. The properties for each case show on Table 1.
9.2 Channel Capacity

On this final section of the manual we will display various channel capacity plots, each one showing the capacity’s dependence on a different parameter. These plots use the parameter values of Table 6, unless a parameter is a plot’s independent variable. We will again focus on a single path and the plot results are will be a mean of 20 ‘drops’, except Fig. 30 and 31 which are a simulation of a single ‘drop’. The number of BS and MS antenna elements and their type is noted in each plot.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>20</td>
<td>$M$</td>
<td>20</td>
</tr>
<tr>
<td>$\theta_{BS}$</td>
<td>$0^\circ$</td>
<td>$\theta_{MS}$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$E(\sigma_{AS,BS})$</td>
<td>$8^\circ$</td>
<td>$E(\sigma_{AS,MS})$</td>
<td>$68^\circ$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.15 m</td>
<td>$f_c$</td>
<td>2 GHz</td>
</tr>
<tr>
<td>$\theta_v$</td>
<td>$0^\circ$</td>
<td>$v$</td>
<td>60 km/h</td>
</tr>
<tr>
<td>SNR</td>
<td>15 dB</td>
<td>XPD</td>
<td>8 dB</td>
</tr>
<tr>
<td>$d_s$</td>
<td>$4 \lambda$</td>
<td>$d_e$</td>
<td>$0.4 \lambda$</td>
</tr>
</tbody>
</table>

Table 6 Common parameter values for the capacity plots.
Fig. 30  The simulated channel capacity for the duration of one minute for 4 different MIMO antenna systems and their respective capacity means. Here we use omni antennas at both arrays.

Fig. 31  Here we compare the capacities of an 1x1 SISO versus a 2x2 cross polarized, co-located dipole MIMO system. We notice that using co-located dipoles we increase the capacity without using multi-element arrays.
Fig. 32  A 3-D representation of the capacity versus the antenna elements at the BS and MS. We use omni antennas at both arrays.

Fig. 33  Here we can observe that the capacity increases linearly with the number of antenna elements at both the BS and MS. We use omni antennas at both arrays.
Fig. 34  The channel capacity versus the signal to noise ratio for 4 different MIMO using omni antennas at both arrays.

Fig. 35  The channel capacity of a 10x10 MIMO using omni antennas at the BS and dipole antennas at the MS as a function of the common MS antenna dipole tilt for different XPD values.
Fig. 36 The channel capacity of a 2x2 MIMO as a function of the MS antenna element spacing when the BS antenna element spacing is fixed at $2\lambda$. We use omni antennas at both arrays and the result is the mean of 20 ‘drops’.

Fig. 37 The channel capacity of a 2x2 MIMO as a function of the BS antenna element spacing when the MS antenna element spacing is fixed at $0.4\lambda$. We use omni antennas at both arrays and the result is the mean of 20 ‘drops’.
References


