A Simple FDTD Model for Transient Excitation of Antennas by Transmission Lines

James G. Maloney, Kurt L. Shlager and Glenn S. Smith

Abstract — A simple FDTD model is developed for use with antennas that are fed from transmission lines. The model is especially designed for use with transient excitations, where the incident and reflected waveforms within the transmission line are of interest, and the latter is determined directly in the FDTD calculation. The model is verified for both transmission and reception of transient waveforms by comparison with measured results for a cylindrical monopole antenna with a plane reflector.

I. INTRODUCTION

The finite-difference time-domain (FDTD) method has been used to analyze antennas for transient radiation, examples being cylindrical and conical monopole antennas fed through an image plane by a coaxial transmission line [1]–[3]. When a full model is used, the FDTD grid surrounds the antenna and extends into the coaxial feed line; thus, a complete and very accurate description of the transient response is obtained. This includes the reflected voltage within the feed line for a specified incident voltage and the radiated waveform at any point surrounding the antenna. Of course, the small cell size needed to include the coaxial feed can lead to a very large number of cells (unknowns) for the total space modeled. Sometimes this number can be reduced by exploiting some feature of the geometry, e.g., for rotationally symmetric structures like the monopoles, a two-dimensional grid (r, z) can be used to model the three-dimensional antennas. In cases where such simplifications are not possible, it is very useful to have an approximate yet accurate model for the coaxial feed that does not require the physically small cells.

The simple feed models that have been used in the past include specification of the total voltage across a single FDTD cell and specification of the total voltage across a magnetic frill [4], [5]. In both of these cases, it is the total voltage that is specified at the drive point. This is appropriate for steady-state (time-harmonic) excitation, where quantities like the input impedance are of interest. However, for transient excitation one is often interested in specifying the incident voltage waveform (time-domain) and then calculating the reflected voltage waveform in the feeding transmission line. This information is not directly available from either of the two simple feed models mentioned above. To obtain this information from these models, the time-domain waveforms for both the total voltage and current have to be Fourier transformed and the impedance and voltage reflection coefficients determined in the frequency domain. The inverse Fourier transform of the reflection coefficient then must be convolved with the specified incident voltage waveform (time-domain) to obtain the reflected voltage waveform. Similar operations are required to find the radiated field that is a result of the incident voltage waveform.

II. SIMPLE MODEL FOR FEED REGION

The new, simple model for the feed region will be discussed in the context of the antenna shown in Fig. 1. Here a monopole antenna of height h is fed through an image plane by a coaxial line. It is symmetrically located a distance d from a perfectly conducting plane reflector (width w, height s).

Under the assumption that the electromagnetic field within the coaxial line is TEM, the coaxial feed line can be replaced by a one-dimensional transmission line model that is simply connected to the antenna at the image plane as shown in Fig. 2. The voltage and current on the transmission line are updated in the standard FDTD fashion, i.e.,

\[ I^{n+0.5}(k' + 0.5) = I^{n-0.5}(k' + 0.5) \]
\[ - \left( \frac{1}{Z_o} \right) \left( \frac{v \Delta t}{\Delta z} \right) [v^n(k' + 1) - v^n(k')] \]
\[ Y^{n+1}(k') = \frac{v^n(k')}{Z_o} \left( \frac{v \Delta t}{\Delta z} \right) \times [I^{n+0.5}(k' + 0.5) - I^{n+0.5}(k' - 0.5)] \]  

(1)

(2)

where \( Z_o \) is the characteristic impedance, and \( v \) is the phase velocity for the transmission line.

Notice that the FDTD grid for the transmission line and the FDTD grid surrounding the antenna are attached where \( k' = k_{\text{top}}, k = 0 \); here \( k' \) is the index for cells in the transmission line, and \( k \) is an index (z direction) for cells in the space surrounding the antenna. To update the voltage in the aperture (2) with \( k' = k_{\text{top}} \), the current above the aperture is needed. This is computed from local magnetic
The incident waveform (\(V_{\text{inc}}\)) is introduced into the coaxial line using a "one-way" injector at position \(k' = k'_{\text{source}}\):

\[
\begin{align*}
\mathcal{H}_y^{n+0.5}(k'_{\text{source}} + 0.5) &= \mathcal{H}_y^{n-0.5}(k'_{\text{source}} + 0.5) \\
- \left( \frac{1}{\mu_c} \right) \left( \frac{2 \pi \Delta t}{\Delta x} \right) C_y^0(i_{a} + 0.5, j_{a} + 0.5) &\times V^n(k'_{\text{source}} + 0.5) \\
+ \left( \frac{\pi \Delta f}{\mu_c \Delta x} \right) C_y^0(i_{a} + 1, j_{a} + 0.5) &\times V^n(k'_{\text{source}} + 0.5),
\end{align*}
\]

(6)

\[
\begin{align*}
V^{n+1}(k'_{\text{source}}) &= V^n(k'_{\text{source}}) + \left( \frac{\pi \Delta f}{\mu_c \Delta x} \right) C_y^0(i_{a} + 1, j_{a} + 0.5).
\end{align*}
\]

(7)

This launches a TEM wave within the line in only the +z direction. The bottom of the coaxial line, \(k' = 0\), is truncated with an absorbing boundary condition. The absorbing boundary condition is particularly simple when \(\Delta z / \Delta t = \text{integer}\); for example with \(\Delta z / \Delta t = 2\), it is

\[
V^{n+1}(0) = V^{n-1}(1).
\]

(8)

III. COMPARISON WITH MEASUREMENTS

An experimental model was constructed for the antenna shown in Fig. 1, and various measurements made on the model were compared with FDTD calculations that use the simple feed model. All measurements were made in the frequency domain using a Hewlett-Packard model 8510 automated network analyzer, and the time-domain results were obtained using the fast Fourier transformation.

For all results shown, the cylindrical monopole is excited from the coaxial transmission line (\(Z_o = 50 \Omega\)) by the Gaussian voltage pulse

\[
V'(t) = V_o \exp(-t^2/2\tau_o^2)
\]

(9)

of unit amplitude, \(V_o = 1.0 \text{ V}\), and characteristic time \(\tau_o / \tau_a = 1.61 \times 10^{-1}\), where \(\tau_a = h/c\) is the time required for light to travel the length of the monopole. The radius of the monopole is \(a/h = 0.304\), and the ratio of the radii for the coaxial line is \(h/a = 2.30\). The dimensions and spacing for the plane reflector are \(d/h = 0.75, w/h = 3.4,\) and \(s/h = 2.0\).

The FDTD spatial grid had approximately cubical cells, with the grid increments selected so that \(h / \Delta z = 14, d / \Delta x = 10,\) and \(w / \Delta y = 46.\) The grid extended at least 30 cell increments from the antenna (monopole and/or reflector) in all directions, and it was truncated with a third-order Liao absorbing boundary condition [9].

Fig. 3 is for the monopole antenna alone and shows the reflected voltage within the transmission line and radiated electric field at \(\phi = 0^\circ, \tau = 12.7\). The latter was measured using a small monopole probe, as shown in Fig. 1 and discussed in references [2], [9]. The theoretical results (solid line) and the measured results (solid dots) are seen to be in good agreement. Some of the small differences seen in Fig. 3(b) for the radiated field are due to grid dispersion for the FDTD results; this point is discussed in more detail in Appendix I.
This agreement verifies the simple feed model for the transmitting case.

Fig. 4 is for the monopole antenna with the reflector. The reflected voltage within the transmission line is shown in Fig. 4(a), and the difference between the reflected voltage within the transmission line for the monopole with reflector and without reflector is shown in Fig. 4(b). The latter roughly shows the reception by the monopole of the pulse reflected from the plane, finite reflector. The theoretical results are again in good agreement with the measured results. This agreement verifies the simple feed model for the receiving case.

IV. CONCLUSION

A simple FDTD feed model has been developed for use with antennas that are fed from transmission lines. The model is especially designed for use with transient excitations, where the incident and reflected waveforms within the transmission line are of interest. The model was verified both for transmission and reception by comparison with measured results for a monopole with and without a plane reflector.

Although the discussion in this paper is for a coaxial transmission line, the model can be used with transmission lines of other geometry. In cases where there is substantial direct coupling between the antenna and conductors of the transmission lines, e.g., a dipole fed from a two-wire line with widely spaced conductors, the model may not be appropriate.

APPENDIX I

The grid dispersion inherent in the FDTD method becomes important when propagation is over large distances. In the calculation of the electric field at the probe in Fig. 1, propagation is over the distance \( r = 100\Delta x \), where \( \Delta x \) is the increment for the spatial grid in the direction of propagation. The FDTD grid dispersion associated with propagation over this distance can be calculated from a formula given in [10]. For propagation along the x axis, the total error in phase (Degrees) for a component of the signal at the angular frequency \( \omega \) or the wavelength \( \lambda_o = 2\pi c/\omega \) is approximately

\[
\Theta_{err} = 360^\circ \left( \frac{r}{\Delta x} \right) \left\{ \frac{1}{\pi} \sin^{-1} \left[ 2\sin \left( \frac{\pi \Delta x}{2\lambda_o} \right) \right] - \frac{\Delta x}{\lambda_o} \right\} \tag{10}
\]

A graph of this function (solid line) is shown in Fig. 5(a) along with a graph of the spectrum for the incident Gaussian pulse (9). Clearly there is substantial grid dispersion at frequencies that are significant within the pulse.

Before the FDTD results for the electric field were plotted in Fig. 3(b), the grid dispersion was removed by using equation (10) in the following manner. The FDTD, time-domain waveform for the electric field at the probe was Fourier transformed. Next, equation (10) was used to correct the phase of each frequency component for the dispersion. The inverse Fourier transform was then applied to obtain the dispersion corrected, time-domain waveform for the
waveform after correction for the dispersion. The need for the correction is evident.

Note that the validity of the simple feed model presented in this paper is independent of the grid dispersion. If a smaller spatial grid increment, Δx, could have been used, then the discussion of dispersion would have been unnecessary.

REFERENCES


