Optimisation of Gardner and Non Data-Aided Early-Late ($\lambda = 1/2$) Timing Error Detection Algorithms

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Abstract

The tracking performance of the Gardner algorithm and the non data-aided early-late ($\lambda = 1/2$) algorithm are made almost jitter-free in the presence of additive white Gaussian noise. For this purpose, the newly developed Combined Tracking and Parallel Search (CTAPS) method has been adopted. Computer simulations show that the optimised algorithms maintain the correct timing error under the conditions that the original algorithms lose tracking. Unlike the bit error rate (BER) performance which is unacceptably high at low to medium signal-to-noise ratios (SNRs) when using the original algorithms, the BER performance of the optimised algorithms is close to the theoretical results. A superior tracking performance, a very fast acquisition time, and a low complexity are the features of the optimised algorithms.

1. Introduction

An essential function performed in a digital receiver is the clock timing synchronisation to detect and correct the error between the transmitter and the receiver clocks. An important class of timing error detectors consists of the Gardner algorithm [1] and the non data-aided early-late ($\lambda = 1/2$) algorithm [2] whose operation is independent of the carrier phase error. This is an advantage over algorithms that have to be operated in a joint implementation with a carrier phase synchroniser [3]. However, the common disadvantage of the above algorithms is the presence of self-noise, which is aggravated by an increase in the loop gain or a decrease in the SNR leading to symbol slips. Eliminating the self-noise without affecting the properties of the original algorithms, i.e. fast acquisition, the requirement of 2 samples per symbol to estimate the timing error, and an operation independent of the carrier phase error is the motivation in writing this paper. The approach taken for this purpose is an extension to the analysis presented in [4, 5].

2. Analysis

We are assuming a system whose output is nominally synchronised data symbols subject to additive white Gaussian noise, where the transmitter and the receiver use Nyquist root raised-cosine filters. With perfect Doppler shift correction, the real component, $p_r(k)$, and the imaginary component, $p_i(k)$, of the sampled matched filter output, $p(k)$, are:

$$p_r(k) = A_s \sum_{n=\infty}^{\infty} a(n)g((k-n)T - \tau)e^{j\theta} + w(k)$$  \(1\)

$$p_i(k) = A_s \sum_{n=\infty}^{\infty} b(n)g((k-n)T - \tau)e^{j\theta} + w(k)$$  \(2\)

Where $A_s$ is the amplitude of the received signal; $a(n)$ and $b(n)$ are the real and the imaginary components of data symbols, respectively; $\tau$ is the timing error; $\theta$ is the carrier phase error; $w(k)$ is the additive white Gaussian noise; and $k$ is the symbol number. The timing error detector output for the QPSK modulation is given by [1]:

$$\hat{\tau}(k) = p_r(k + 1/2)[p_r(k) - p_r(k + 1)] + p_i(k + 1/2)[p_i(k) - p_i(k + 1)]$$  \(3\)

Non data-aided early-late (NDAEL) algorithm, $\lambda = 1/2$ is given by [2]:

$$\hat{\tau}(k) = p_r(k)[p_r(k + 1/2) - p_r(k - 1/2)] + p_i(k)[p_i(k + 1/2) - p_i(k - 1/2)]$$  \(4\)

By substituting $p_i(k) = 0$ in (3) and (4), the corresponding algorithms for the BPSK modulation...
scheme result. By investigation, the following general expression for the above algorithms can be deduced:

\[ \hat{\tau}(k) = (p_r(k) - Zp_r(k + 1)) \times \]

\[ \{p_r(k + 1/2) - (1 - Z)p_r(k - 1/2)\} \]

\[ + (p_r(k) - Zp_r(k + 1)) \times \]

\[ \{p_r(k + 1/2) - (1 - Z)p_r(k - 1/2)\} \]

where \( Z = 1 \) for the Gardner algorithm, and \( Z = 0 \) for the non data-aided algorithm. By substituting (1) and (2) in (5), the generalised algorithm can be expanded into the following manageable terms:

\[ \hat{\tau}(k) = \hat{\tau}_{n=k} + \hat{\tau}_{n=k+1} + \hat{\tau}_{n=k-1} + \hat{\tau}_{n=k,k \pm 1} + w(k) \]  

where

\[ \hat{\tau}_{n=k} = A^2 e^{j2\theta} [g(\tau) - Zg(T + \tau)] \]

\[ \times [g(\tau - T/2 - \tau)] - (1 - Z)g(-3T/2 - \tau)] \]

\[ \times [a^2(k) + b^2(k)] \]  

\[ \hat{\tau}_{n=k+1} = A^2 e^{j2\theta} [g(-T + \tau) - Zg(-\tau)] \]

\[ \times [g(-T/2 - \tau)] - (1 - Z)g(-3T/2 - \tau)] \]

\[ \times [a^2(k + 1) + b^2(k + 1)] \]  

\[ \hat{\tau}_{n=k-1} = A^2 e^{j2\theta} [g(T - \tau) - Zg(2T - \tau)] \]

\[ \times [g(3T/2 - \tau)] - (1 - Z)g(-3T/2 - \tau)] \]

\[ \times [a^2(k - 1) + b^2(k - 1)] \]  

\[ \hat{\tau}_{n=k,k \pm 1} = A^2 e^{j2\theta} \sum_{n=\pm 1}^{\infty} g[(k + n)]T \]

\[ - Zg([(k + n)]T - \tau)] \]

\[ \times [g([(k + n)]T - \tau)] - (1 - Z)g([-n]T - \tau)] \]

\[ \times [a^2(n) + b^2(n)] \]  

\[ g(t) \] is impulse response of the entire communications system. With Nyquist pulse shaping and \( \tau \neq 0 \), the term \( \hat{\tau}_{n=k} \) is the useful contribution to a DC tracking term. The terms with \( \hat{\tau}_{n=k+1} \) and \( \hat{\tau}_{n=k-1} \) are the self-noise due to the adjacent pulses, and all the remaining terms are the contribution to the self-noise due to the other pulses. When \( \tau = 0 \), as in the case of perfect synchronisation, for the Gardner algorithm the terms \( \hat{\tau}_{jok} \) and \( \hat{\tau}_{jok, k \pm 1} \) are non zero and give rise to the self-noise of the algorithm. For the case of the non data-aided early-late algorithm, all the terms are zero and, hence, the algorithm is in theory self-noise free. However, in practice there is a self-noise in the algorithm due to \( g(T/2) \) not being exactly equal to \( g(-T/2) \). Similarly, in practice \( g(kT) \) is not exactly equal to zero when \( k \neq 0 \). The average is correct, but any individual points can depart from the average giving rise to the self-noise of the algorithm. Self-noise can degrade the performance of the receiver to an extent that the BER may become unacceptable.

3. Minimisation of the Self-Noise

A method to reduce the self-noise is to increase the rolloff factor \( \alpha \) of the Nyquist pulse shape. By simulating the algorithms given in (3) and (4) become self-noise free when \( \alpha = 100\% \). This choice of the rolloff factor is not of interest in practical applications because of the extra bandwidth required to transmit the data. This means that since lower values of \( \alpha \) are used in practice, self-noise will be present.

A second method to reduce the effect of the self-noise is to decrease the loop gain. The acquisition and tracking behaviour of the loop is controlled by the loop gain factor \( \beta \), and loop bandwidth. For a fixed loop bandwidth, higher values of \( \beta \) result in a faster acquisition at the expense of larger tracking jitter. On the other hand as \( \beta \) decreases, the tracking jitter decreases but the acquisition time increases. As an example, the root-mean-square (RMS) of the self-noise for the case of the Gardner algorithm in (3) changes as [3]:

\[ \sigma = 0.1 \beta \]  

for a loop filter consisting of a simple integrator. From the above discussion, it can be deduced that there is a tradeoff between the desired performance and the chance of the parameters \( \alpha \) and \( \beta \). Ideally, it is desirable to

1. keep \( \alpha \) to typical values, such as 0.4—0.65, which are of interest in applications such as those in satellite communications, and
2. choose \( \beta \) to the highest value to achieve the fastest possible acquisition and the lowest possible jitter.

Satisfying the second criteria is a challenge in designing modems. The key to meeting this requirement is to explore ways to cancel the self-noise.
3. Combined Tracking and Parallel Search (CTAPS) Method

Figure 1. Simplified block diagram of the generalised implementation of almost jitter-free timing error synchronisation.

By smoothing the timing estimate \( \hat{\tau}(k) \) by a simple integrator and scaling the smoothed output by the loop gain factor \( \beta \), the trial timing error \( \tilde{\tau}(k) \) is obtained. Assuming there are no symbol slips, the trial error during tracking is equal to a DC value which has been masked by jitter, i.e. \( \tilde{\tau}(k) = \text{DC value} + \text{jitter} \) [3]. This DC value represents the timing error \( \tau \). In order to cancel the jitter and extract the DC value, a combined tracking and parallel search (CTAPS) algorithm is implemented as shown in Figure 1. In the search block, the range \([-T/2, T/2]\) is divided into \( L = T/T_s + 1 \) equally spaced levels, i.e.

\[
\tau_1 = -\frac{T}{2}, \quad \tau_2 = \frac{T}{2} + T_s, \quad \tau_3 = \frac{T}{2} + 2T_s, \ldots, \tau_L = \frac{T}{2}
\]

where \( T_s \) is the sampling period. Sample values \( \tilde{\tau}(k) \) and \( \tilde{\tau}(k-1) \) are compared with the values of \( \tau_i \), where \( \tau_i \) is any of the above levels. For speed of operation, the comparison units must be implemented in parallel. If the sample values traverse the value of \( \tau_i \), then a crossing is said to occur. It is only at one of the values of \( \tau_i \) that the running sum of crossings of \( \tilde{\tau}(k) \) is maximum. This \( \tau_i \) is the desired DC which represents \( \tau \) and is used by the combined matched filter and interpolator to determine the optimum sampling time of the input baseband signal.

Now we assume a symbol slip occurs. The new trial value of the timing error is \( \tau + \text{jitter} \pm T \). Since with the CTAPS method, the search is carried out in the range \([-T/2, T/2]\), the timing error tracked before the symbol slip is still maintained. Therefore, the CTAPS method is immune to the effect of symbol slips.

The above method requires \( L \) comparisons per \( \tilde{\tau}(k) \).

Since in practice \( T/T_s \) is chosen to be two or four [3], the hardware complexity is small and, as will be shown shortly, is outweighed by the superior performance achieved.

4. Simulation Results

Figure 2: Simulation results of the original and optimised Gardner algorithms.

A QPSK modem was simulated and the technique described was incorporated in the modem which was using the Gardner algorithm given in (3).

Figure 2(a) shows the timing error synchronised with an initial error of $3T/8$, signal to noise ratio (SNR) of $2 \text{ dB}$, and loop gain factor $\beta = 0.75$. It can be seen that although the original algorithm has acquired the error, the tracking jitter is high. Furthermore, the algorithm shows the undesirable feature of symbol slips. Figure 2(b) shows the performance of the Gardner algorithm with the optimisation method described; not only has the tracking performance become almost jitter free, but there are also no symbol slips. In order to emphasize the fast acquisition, the results in Figure 2(a) have been shown over the first 200 symbols in Figure 2(b). The acquisition time is about 40 symbols. Such a fast acquisition time is an attractive feature in applications such as those for low earth orbit (LEO) satellites [3].

The BER performance of the simulated modem has been shown in Figure 2(c). To simulate the BER of the modem at a given SNR, 500 errors have been counted and the results have been compared against the theoretical probability of error [7]:

$$P_e = 0.5 \text{erfc} \left( \sqrt{\frac{E_b}{N_o}} \right)$$

(12)

where $E_b$ is the energy per bit, $N_o$ is the one-sided noise spectral density, and $\text{erfc}(x)$ is the complementary error function of $x$. Due to high tracking jitter exacerbated by symbol slips, the receiver’s decision errors at low-to-medium SNRs are high and, hence, the error rate is high. Such an error rate is unacceptable. On the same figure, the error rate of the optimised algorithm is shown. The factors contributing to the superior BER performance of the optimised algorithm are:

1- almost jitter-free tracking performance, and
2- no symbol slips even over long observation periods.

Therefore the optimisation technique presented in attractive for consideration in practical applications. Similar tracking performance and BER have been obtained using the non data-aided algorithm given in (4). Further simulations have been carried out with the BPSK modulation scheme under different SNRs, loop gain factors, and timing accuracies. The results are consistently better than those obtained with the original algorithms.

5. Conclusions

In this paper, the Combined Tracking and Parallel Search (CTAPS) method was presented to make the tracking performance of the Gardner algorithm and the non data-aided early-late ($\lambda = 1/2$) algorithm almost jitter-free. No only has the tracking jitter been greatly reduced, but there are also no symbol slips. These two factors contributed to the BER performance which was close to the theoretical results. A fast acquisition time, a superior tracking performance and good BER performance make our technique suitable for consideration in digital receivers.

6. References


